

Lifetime lost from time series of counts

A convolution formulation of mortality displacement

Jonas Schöley



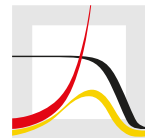
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Everything in mortality is a tempo effect

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Displaced deaths

But they may only have lived for another month anyway...

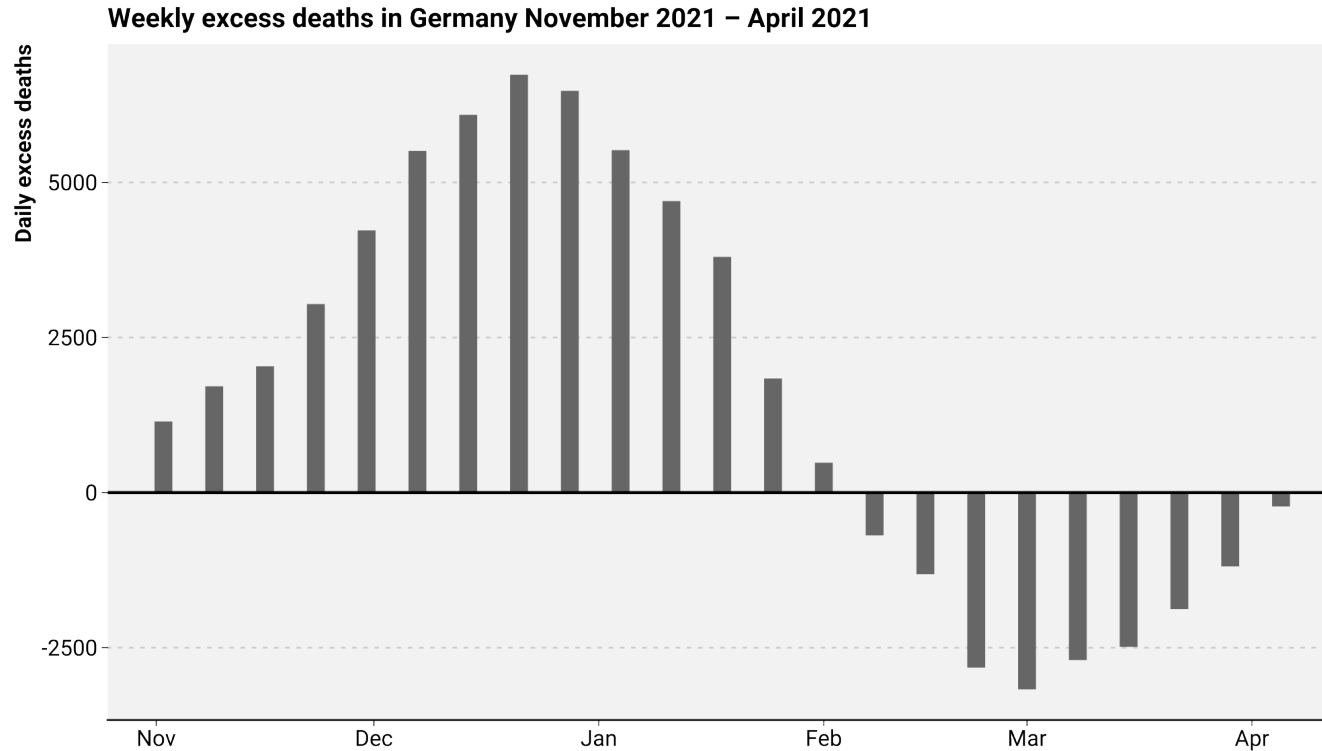
Displaced deaths

But they may only have lived for another month anyway...



Displaced deaths

But they may only have lived for another month anyway...

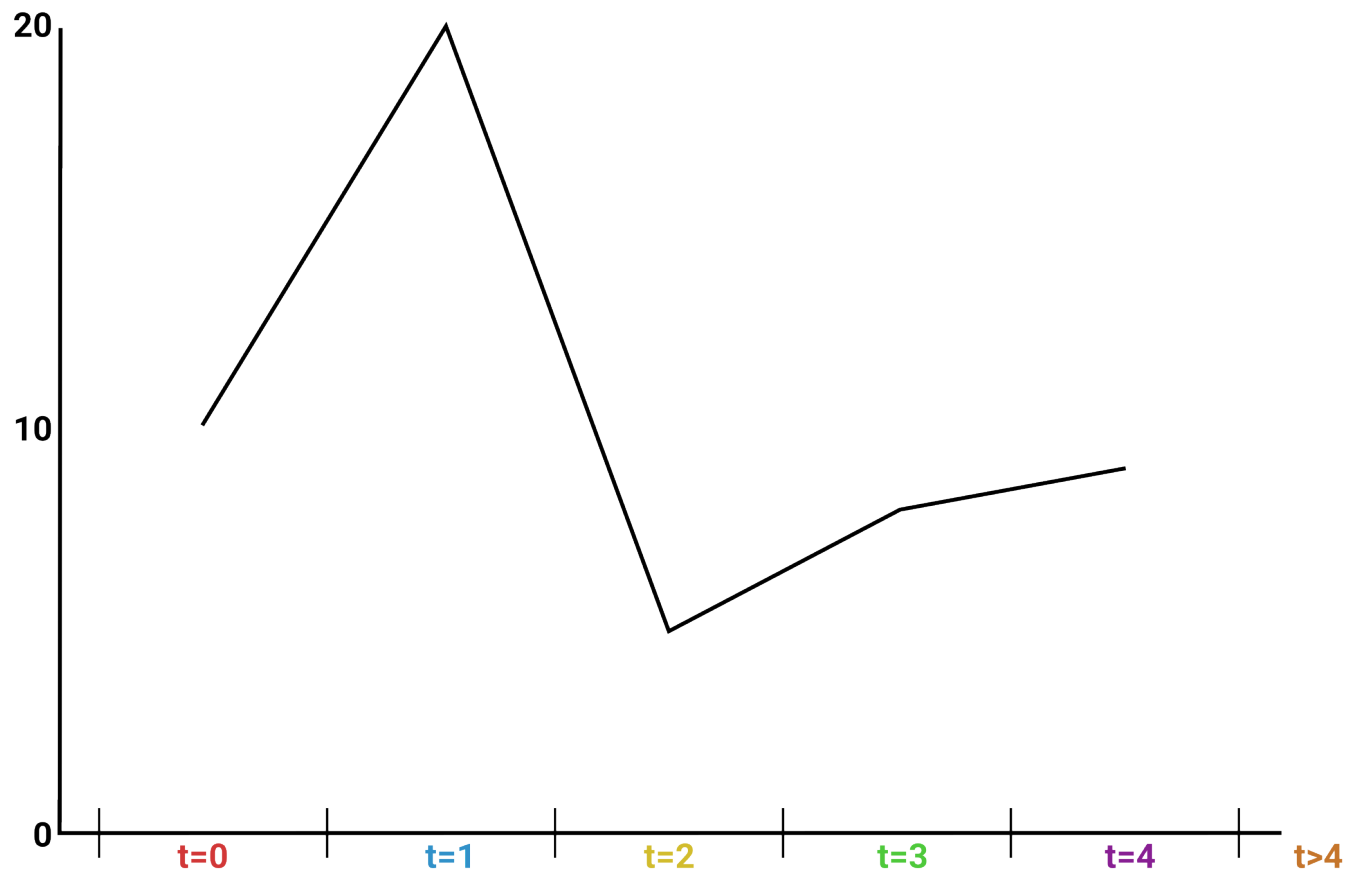


Observed deaths are expected deaths **displaced in time**

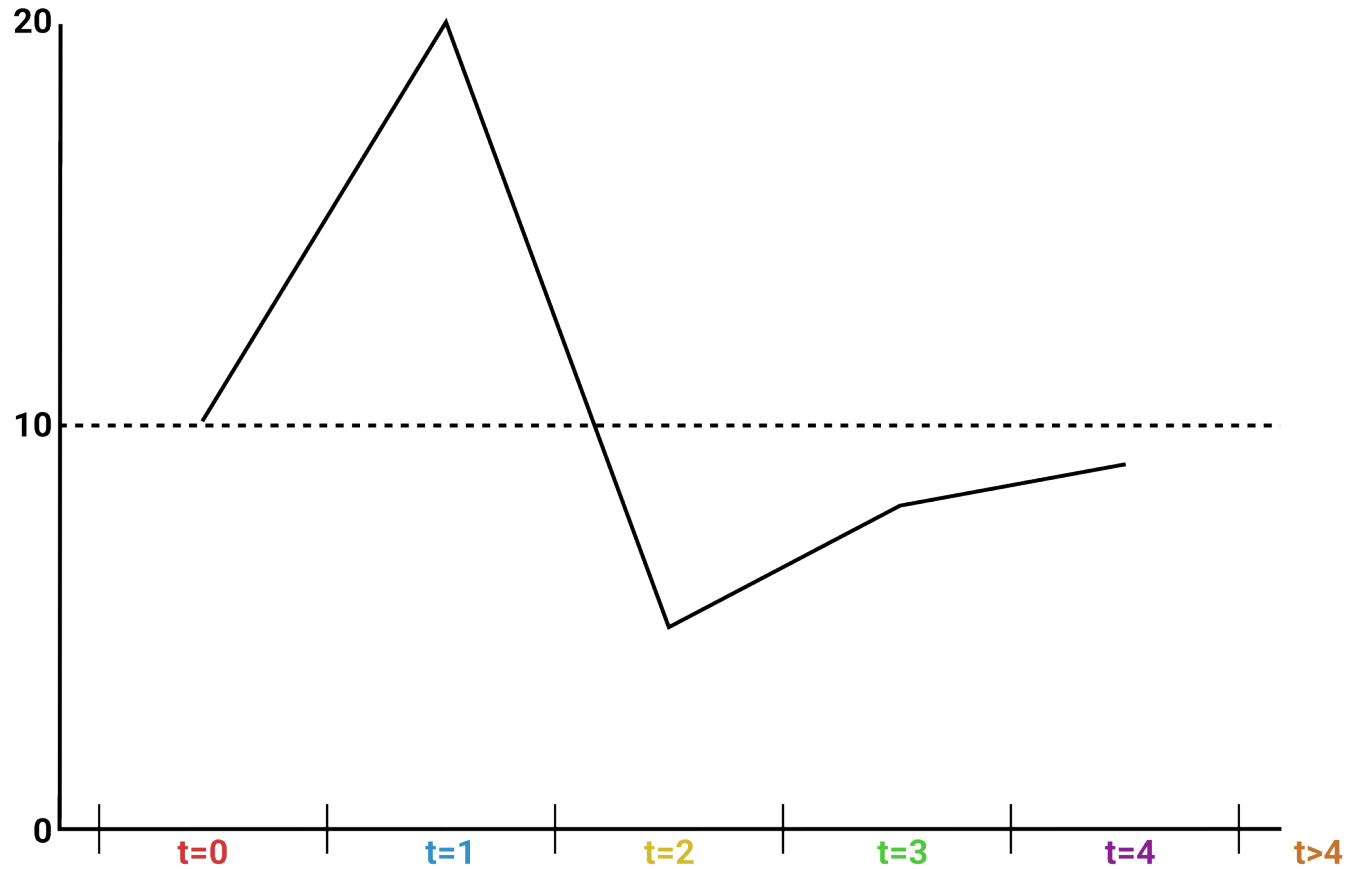
aka

In mortality there are only **tempo effects**

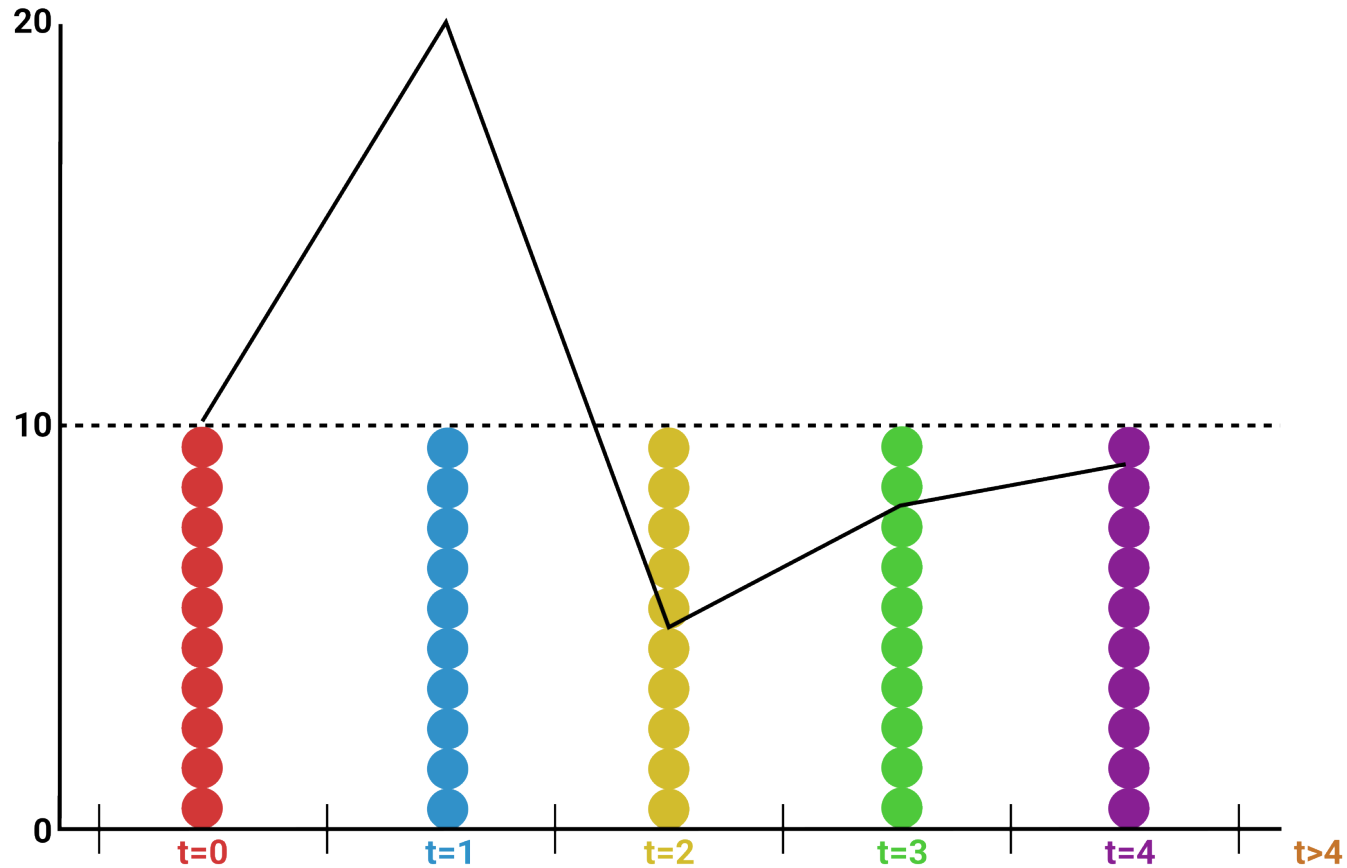
Displaced deaths



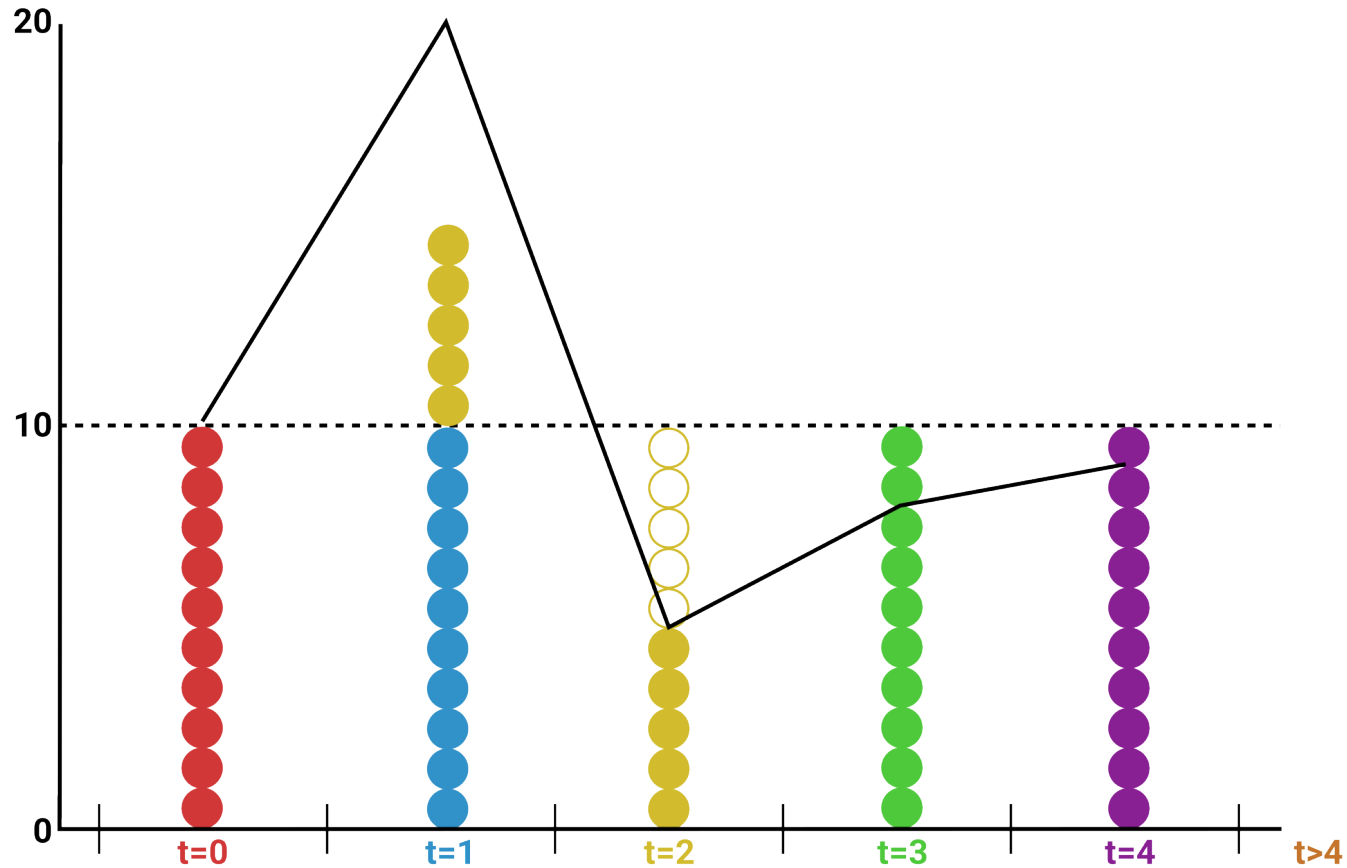
Displaced deaths



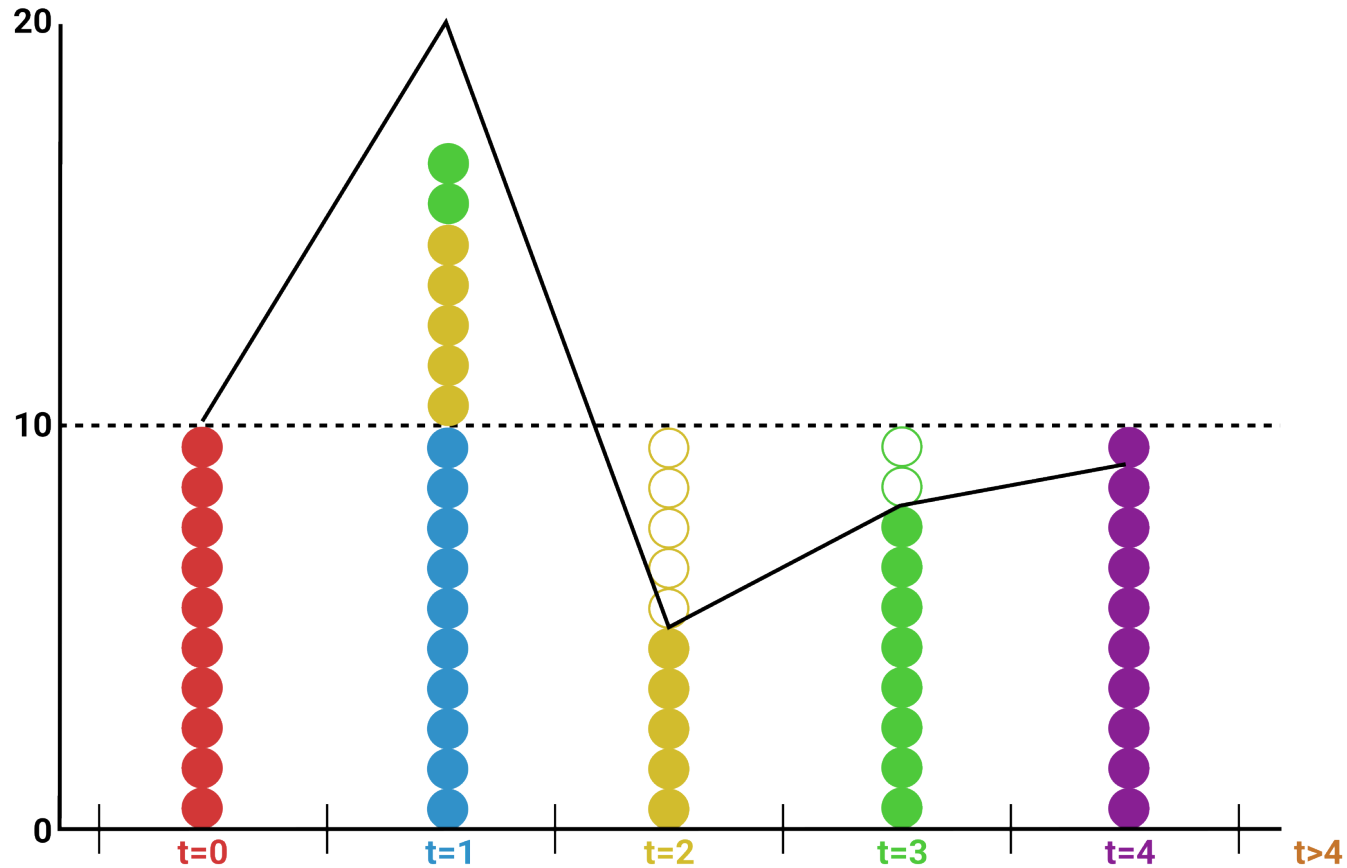
Displaced deaths



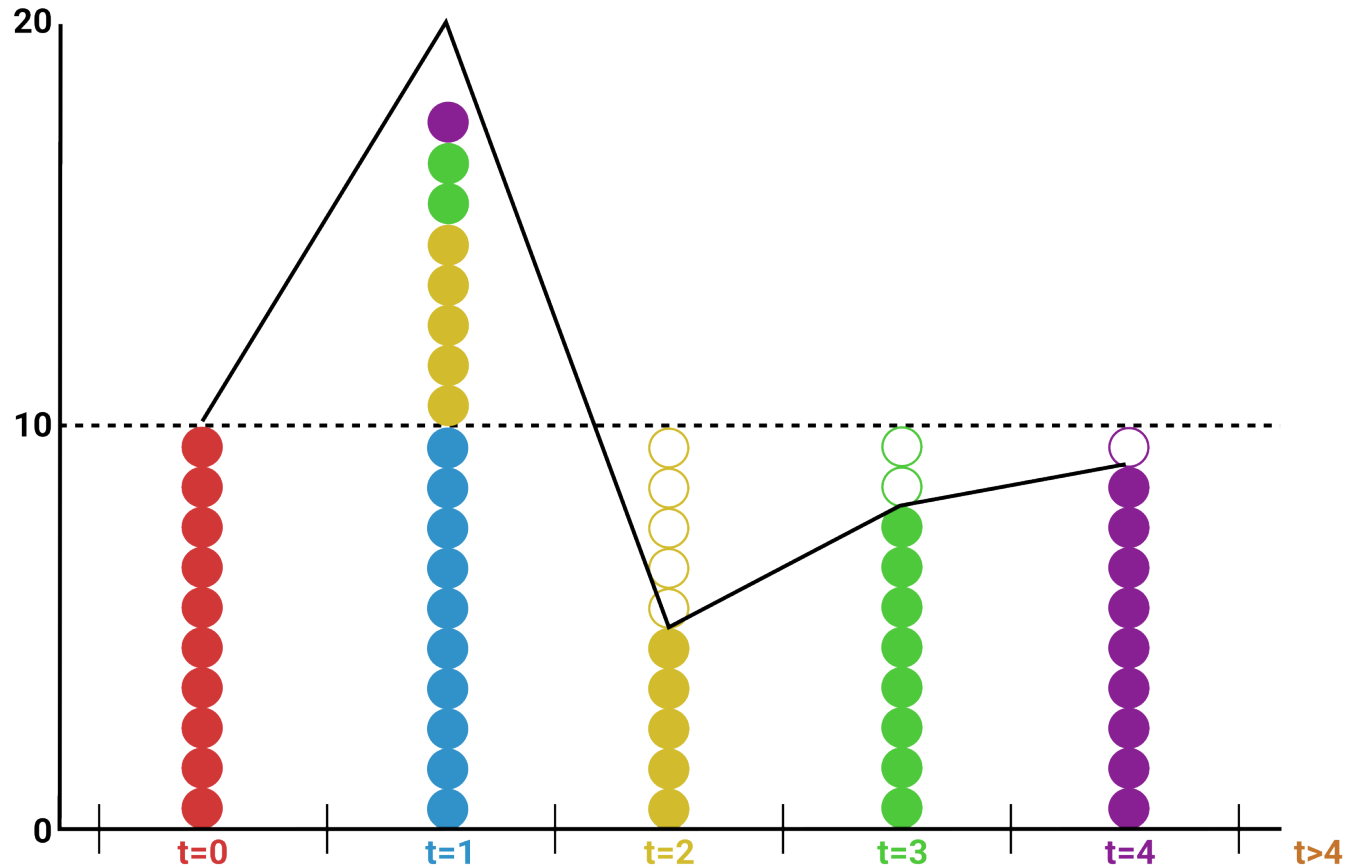
Displaced deaths



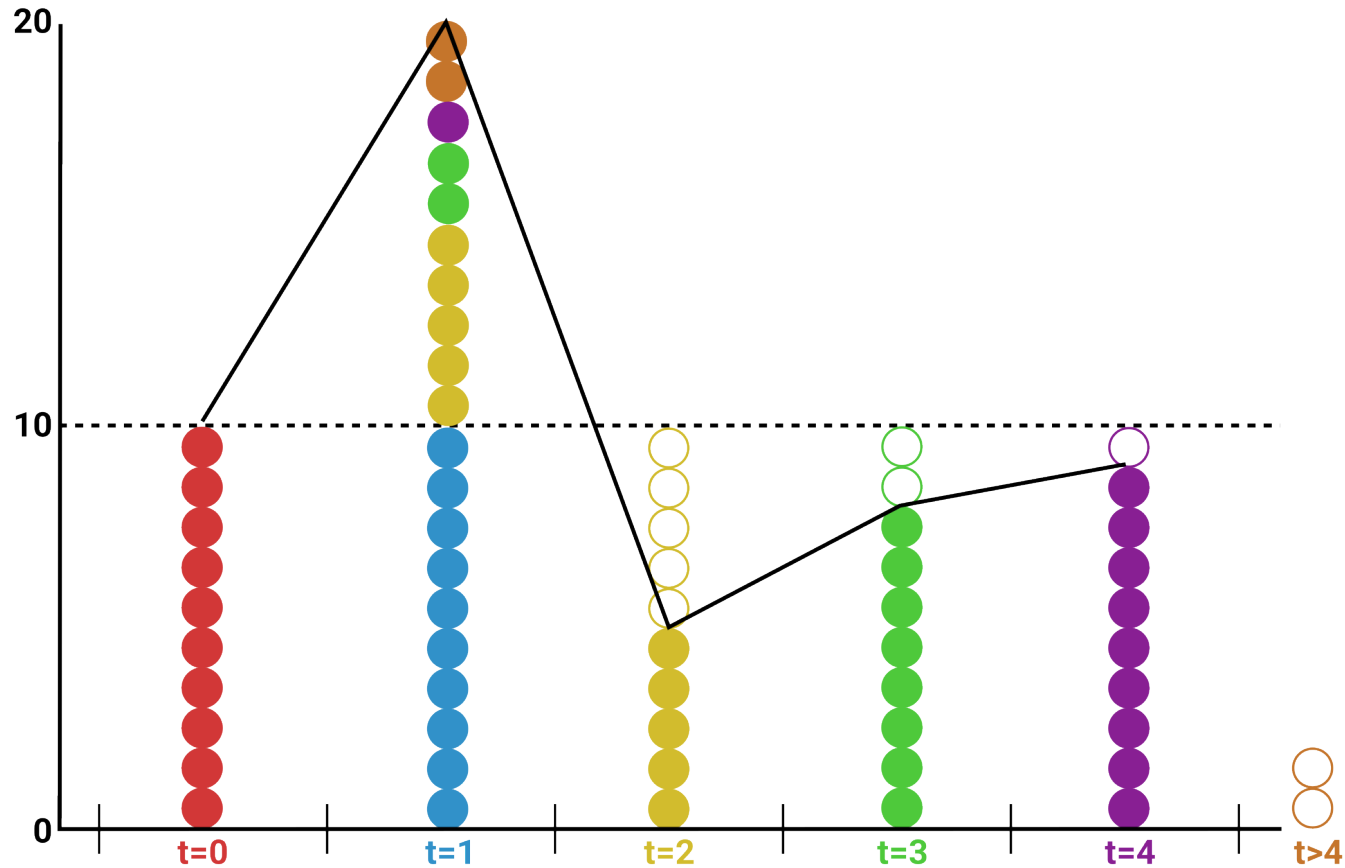
Displaced deaths



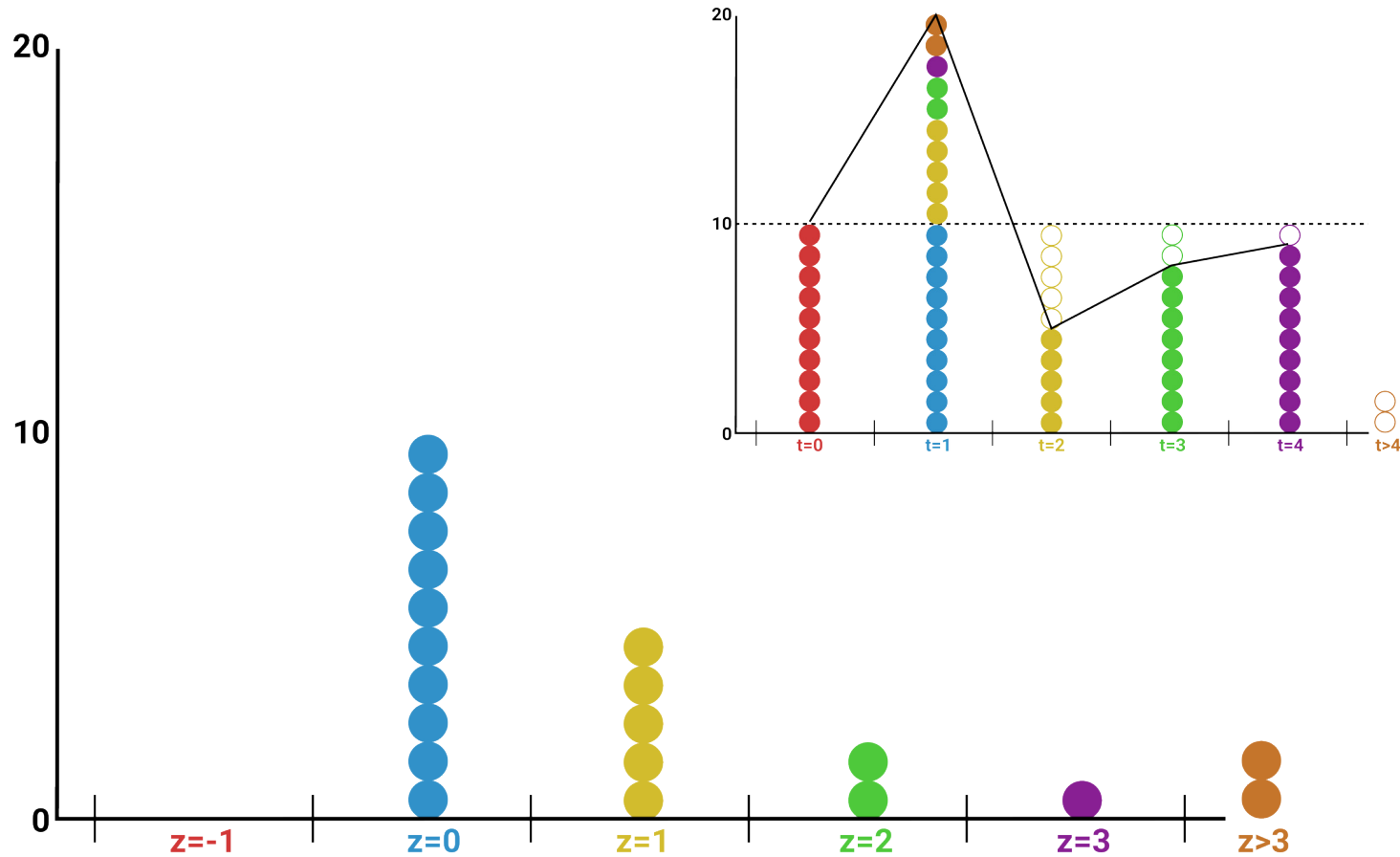
Displaced deaths



Displaced deaths



Displaced deaths

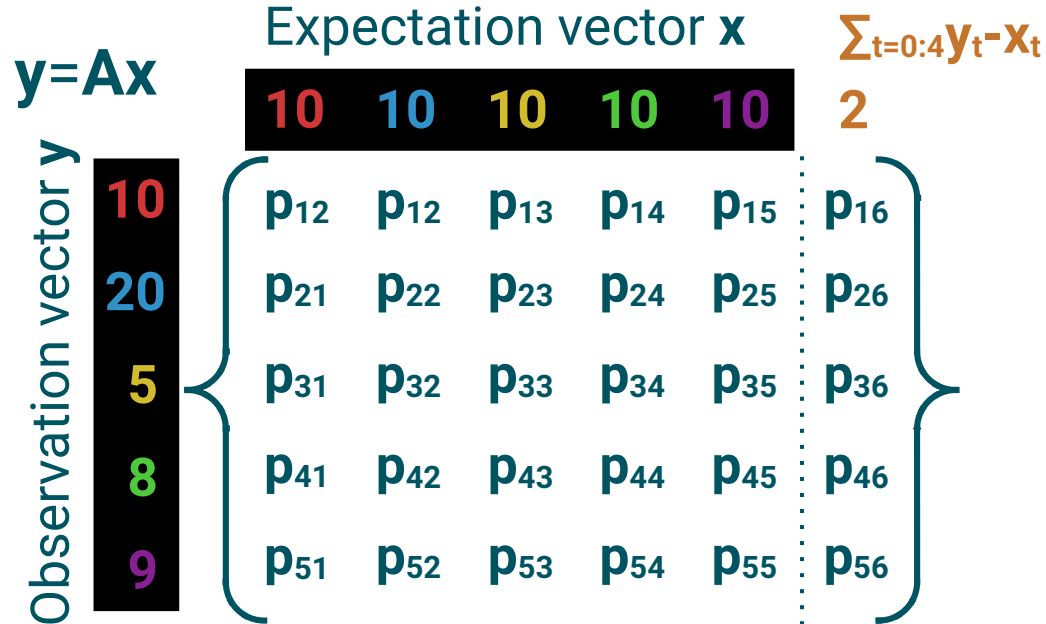


The displacement matrix

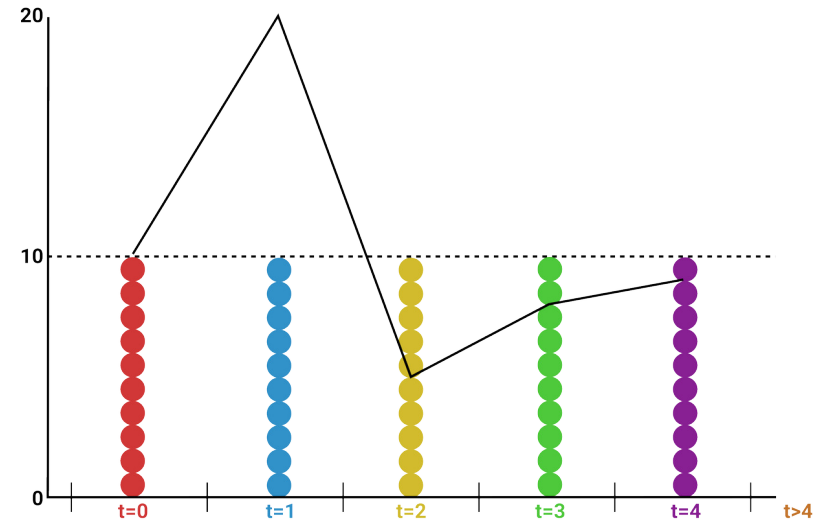
Observed deaths are expected deaths **displaced** in time

$$y = Ax$$

The displacement matrix



$$p = N^2 + N$$

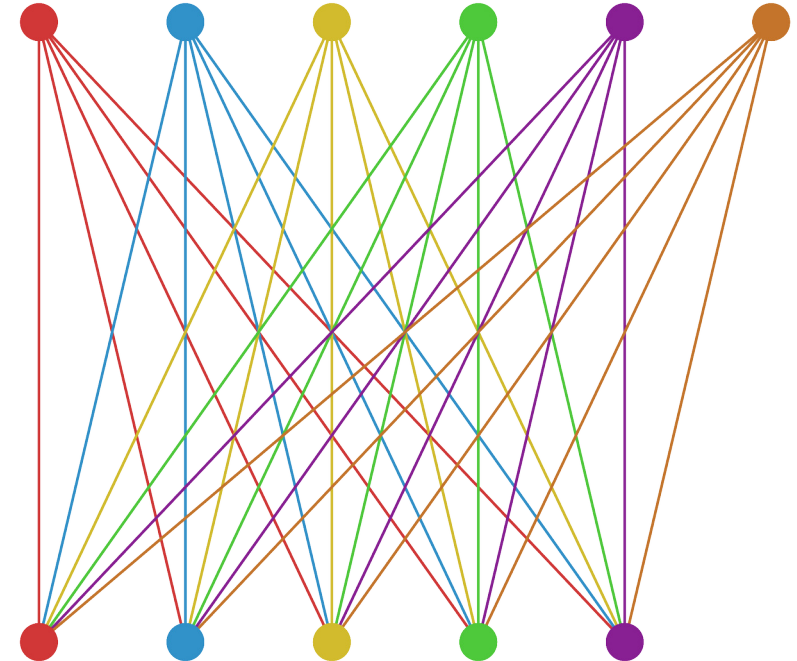


The displacement matrix

$$y = Ax$$

	10	10	10	10	10	$\sum_{t=0:4} y_t - x_t$ 2
10	p_{12}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
20	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}
5	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}
8	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}
9	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}

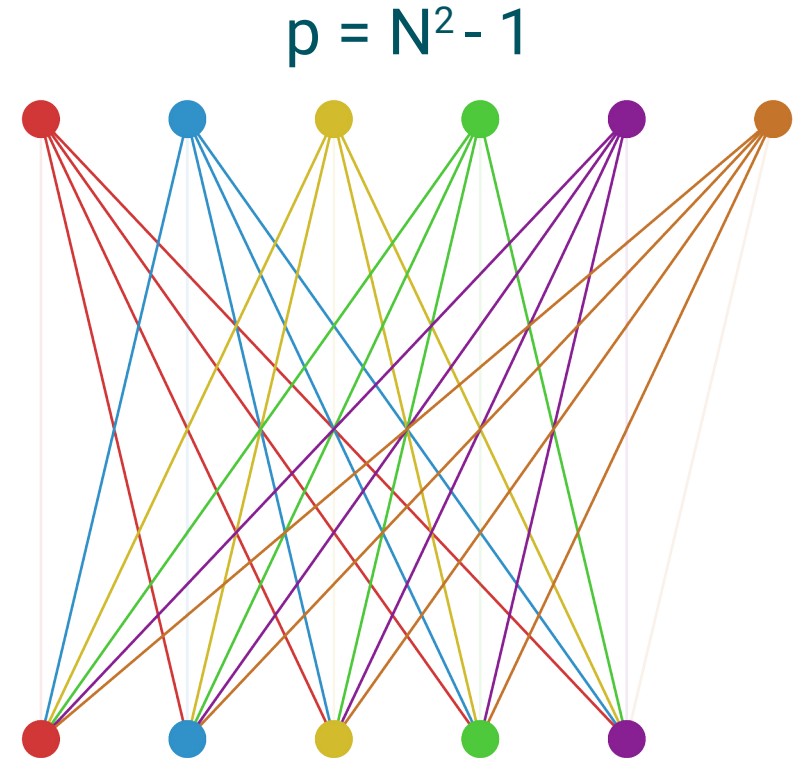
$$p = N^2 + N$$



The displacement matrix

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

$$\begin{array}{c}
 \mathbf{10} \\
 \mathbf{20} \\
 \mathbf{5} \\
 \mathbf{8} \\
 \mathbf{9}
 \end{array}
 \left\{
 \begin{array}{cccccc}
 \mathbf{10} & \mathbf{10} & \mathbf{10} & \mathbf{10} & \mathbf{10} & \mathbf{2} \\
 1 - \sum_i p_{i1} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\
 p_{21} & 1 - \sum_i p_{i2} & p_{23} & p_{24} & p_{25} & p_{26} \\
 p_{31} & p_{32} & 1 - \sum_i p_{i3} & p_{34} & p_{35} & p_{36} \\
 p_{41} & p_{42} & p_{43} & 1 - \sum_i p_{i4} & p_{45} & p_{46} \\
 p_{51} & p_{52} & p_{53} & p_{54} & 1 - \sum_i p_{i5} & 1 - \sum_i p_{i6}
 \end{array}
 \right\}
 \sum_{t=0:4} \mathbf{y}_t - \mathbf{x}_t$$



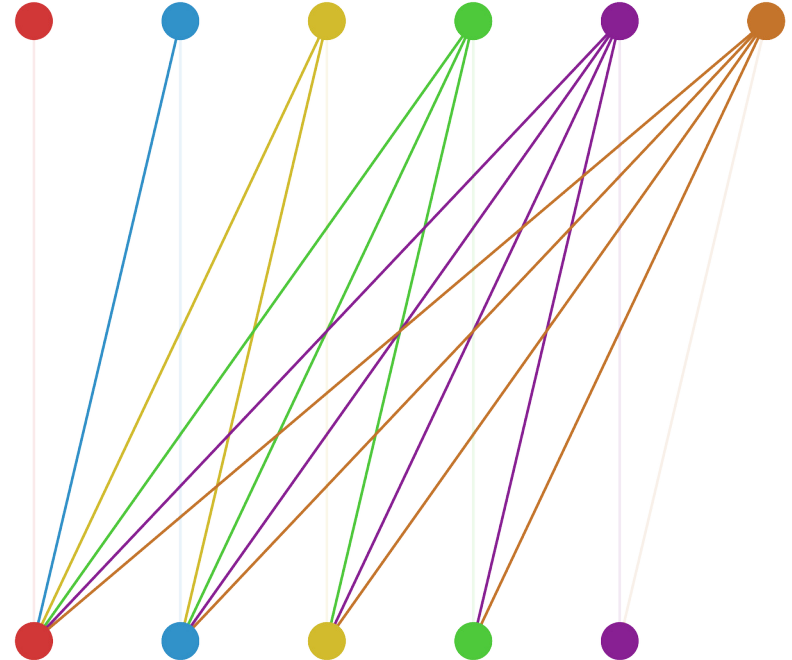
Pure displacement

The displacement matrix

$$y = Ax$$

$$\begin{array}{c}
 \text{10} \\
 \text{20} \\
 \text{5} \\
 \text{8} \\
 \text{9}
 \end{array}
 \left\{
 \begin{array}{cccccc}
 \text{10} & \text{10} & \text{10} & \text{10} & \text{10} & \text{2} \\
 1 - \sum_i p_{i1} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\
 0 & 1 - \sum_i p_{i2} & p_{23} & p_{24} & p_{25} & p_{26} \\
 0 & 0 & 1 - \sum_i p_{i3} & p_{34} & p_{35} & p_{36} \\
 0 & 0 & 0 & 1 - \sum_i p_{i4} & p_{45} & p_{46} \\
 0 & 0 & 0 & 0 & 1 - \sum_i p_{i5} & 1 - \sum_i p_{i6}
 \end{array}
 \right\}
 \sum_{t=0:4} y_t - x_t$$

$$p = 1/2(N + 2)(N - 1)$$

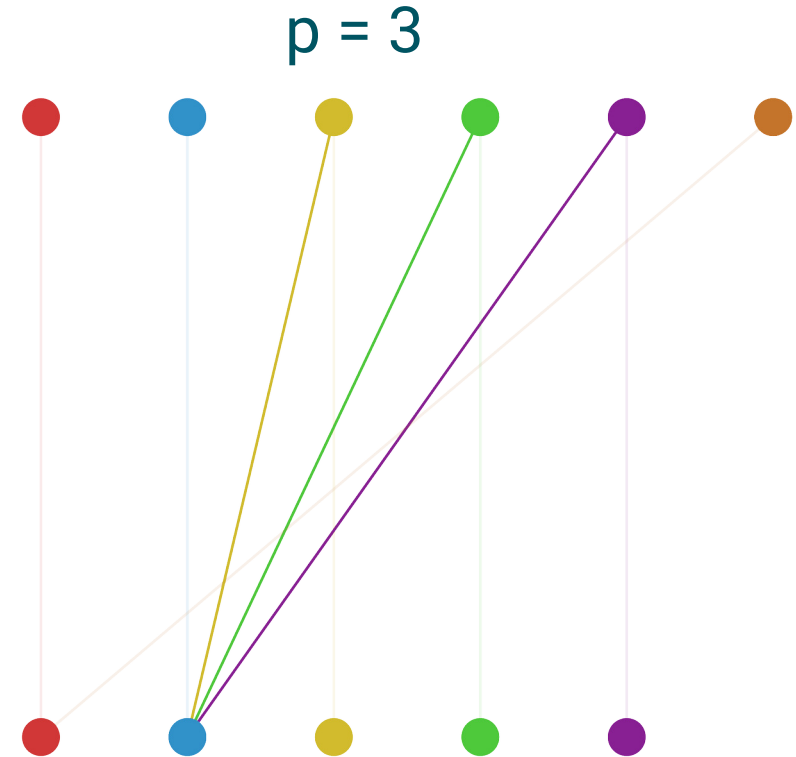


Pure displacement + No lifesaving

The displacement matrix

$$y = Ax$$

$$\begin{array}{c}
 \text{10} \\
 \text{20} \\
 \text{5} \\
 \text{8} \\
 \text{9}
 \end{array}
 \left\{
 \begin{array}{cccccc}
 \text{10} & \text{10} & \text{10} & \text{10} & \text{10} & \sum_{t=0:4} y_t - x_t \\
 1 - \sum_i p_{i1} & 0 & 0 & 0 & 0 & 2 \\
 0 & 1 - \sum_i p_{i2} & p_{23} & p_{24} & p_{25} & 0 \\
 0 & 0 & 1 - \sum_i p_{i3} & 0 & 0 & 1 \\
 0 & 0 & 0 & 1 - \sum_i p_{i4} & p_{45} & 0 \\
 0 & 0 & 0 & 0 & 1 - \sum_i p_{i5} & 0
 \end{array}
 \right\}$$



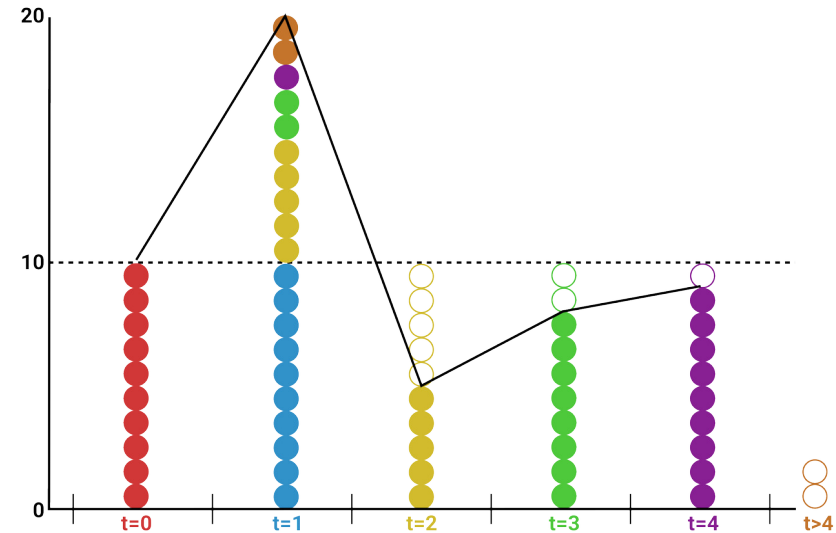
Pure displacement + No lifesaving + Life lost only during excess

The displacement matrix

$$y = Ax$$

		10	10	10	10	10	$\sum_{t=0:4} y_t - x_t$
		10					2
10	{	1	0	0	0	0	}
20		0	1	.5	.2	.1	
5		0	0	.5	0	0	
8		0	0	0	.8	0	
9		0	0	0	0	.9	
							0

$p = 3$

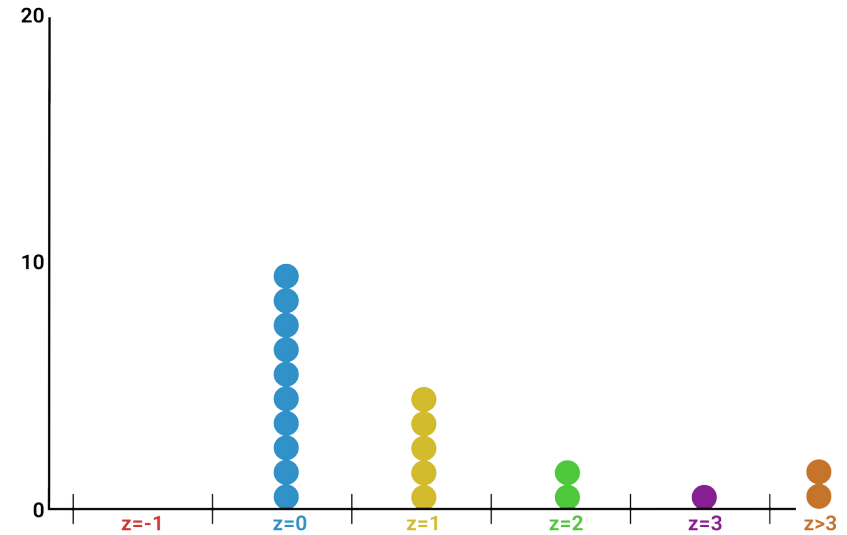


Pure displacement + No lifesaving + Life lost only during excess

The displacement matrix

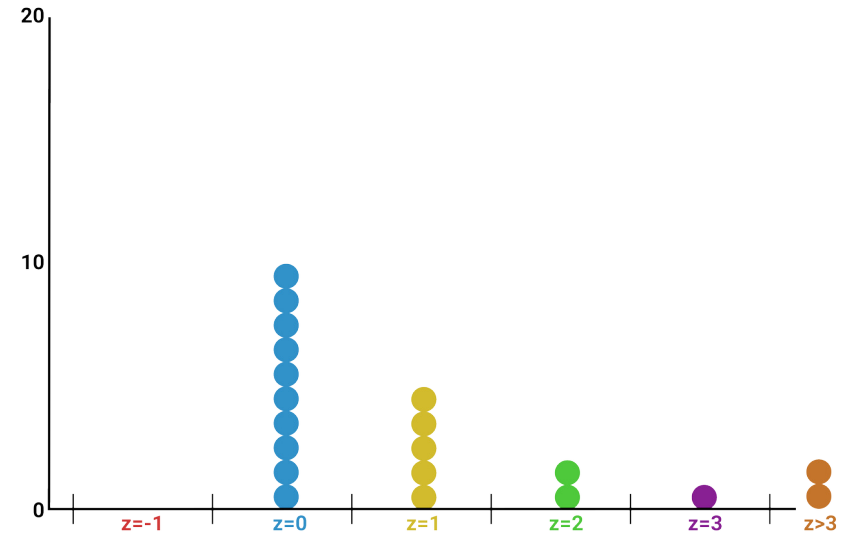
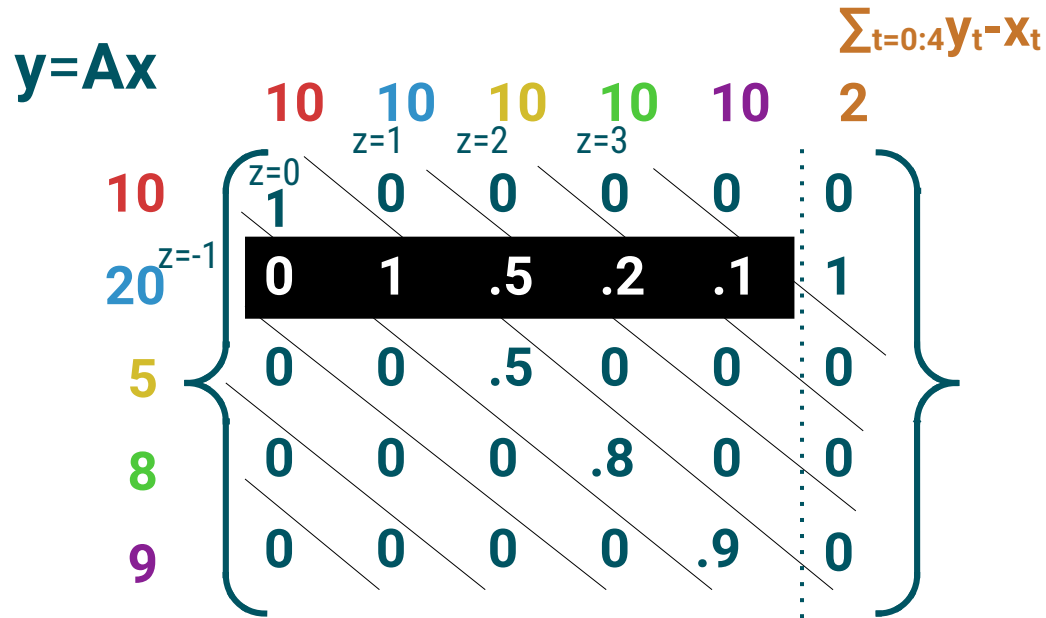
$y = Ax$

$\begin{matrix} 10 \\ 20 \\ 5 \\ 8 \\ 9 \end{matrix}$
 $\left\{ \begin{array}{ccccc|c} 10 & 10 & 10 & 10 & 10 & \sum_{t=0:4} y_t - x_t \\ 2 & & & & & \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & .5 & .2 & .1 & 1 \\ 0 & 0 & .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & .8 & 0 & 0 \\ 0 & 0 & 0 & 0 & .9 & 0 \end{array} \right\}$



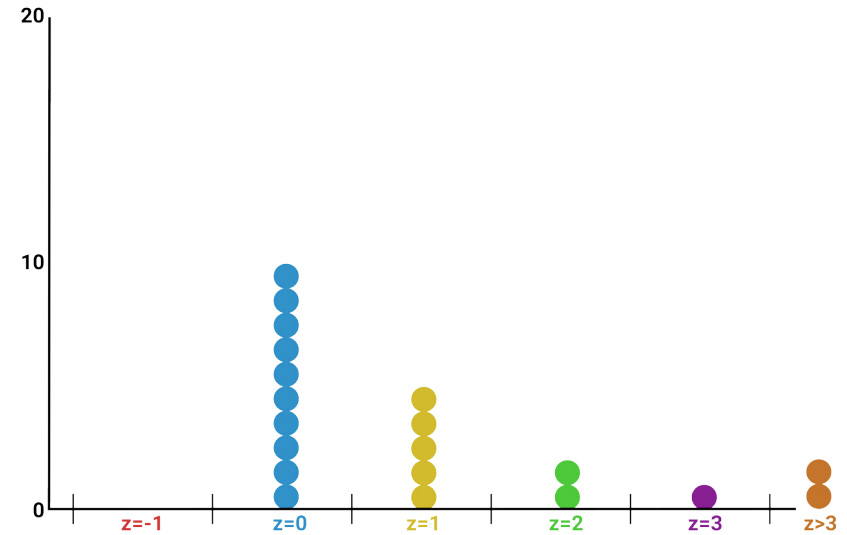
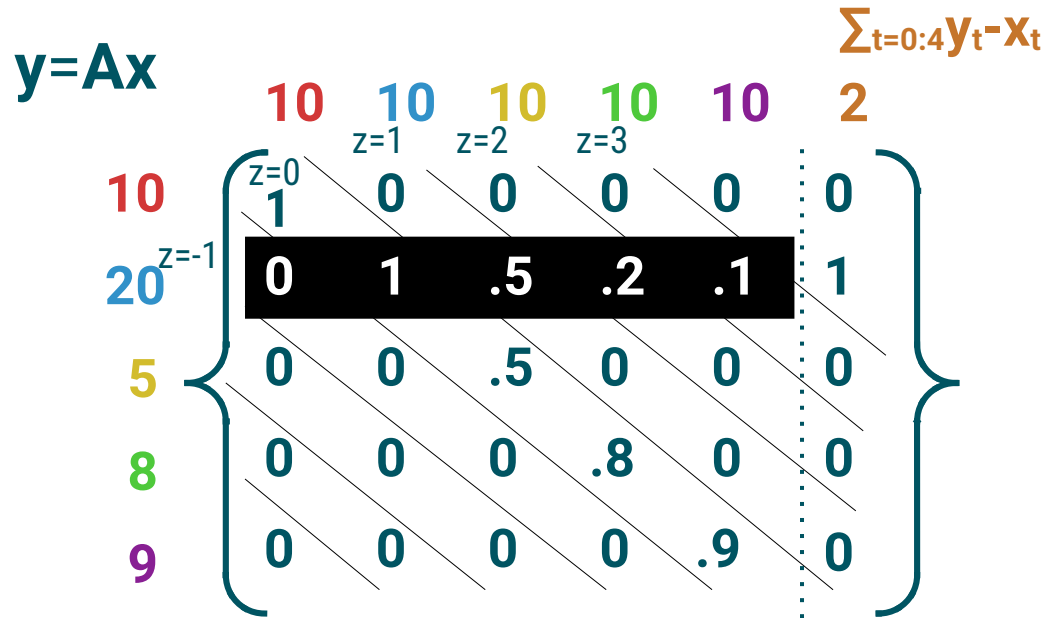
$$y_i = \sum_{j \text{ in } 1:6} x_j a_{ij}$$

The displacement matrix



$$y_i = \sum_{j \text{ in } 1:6} x_j a_{ij}$$

The displacement matrix

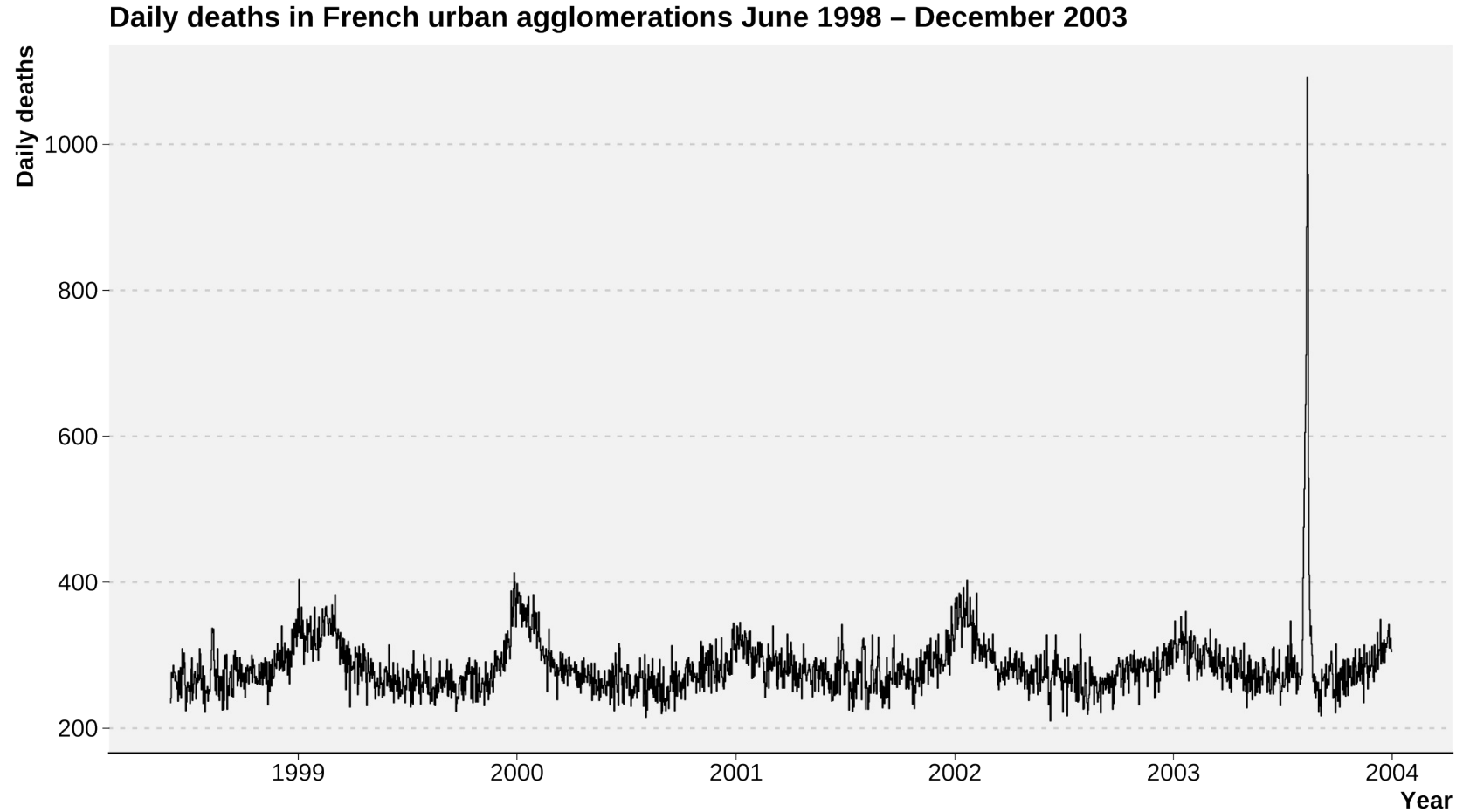


$$y_i = \sum_{j \text{ in } 1:6} x_j a_{ij}$$

$$y_t = \sum_{z \text{ in } -1:4} x_{t+z} a_{t,t+z}$$

Mortality displacement during heatwaves

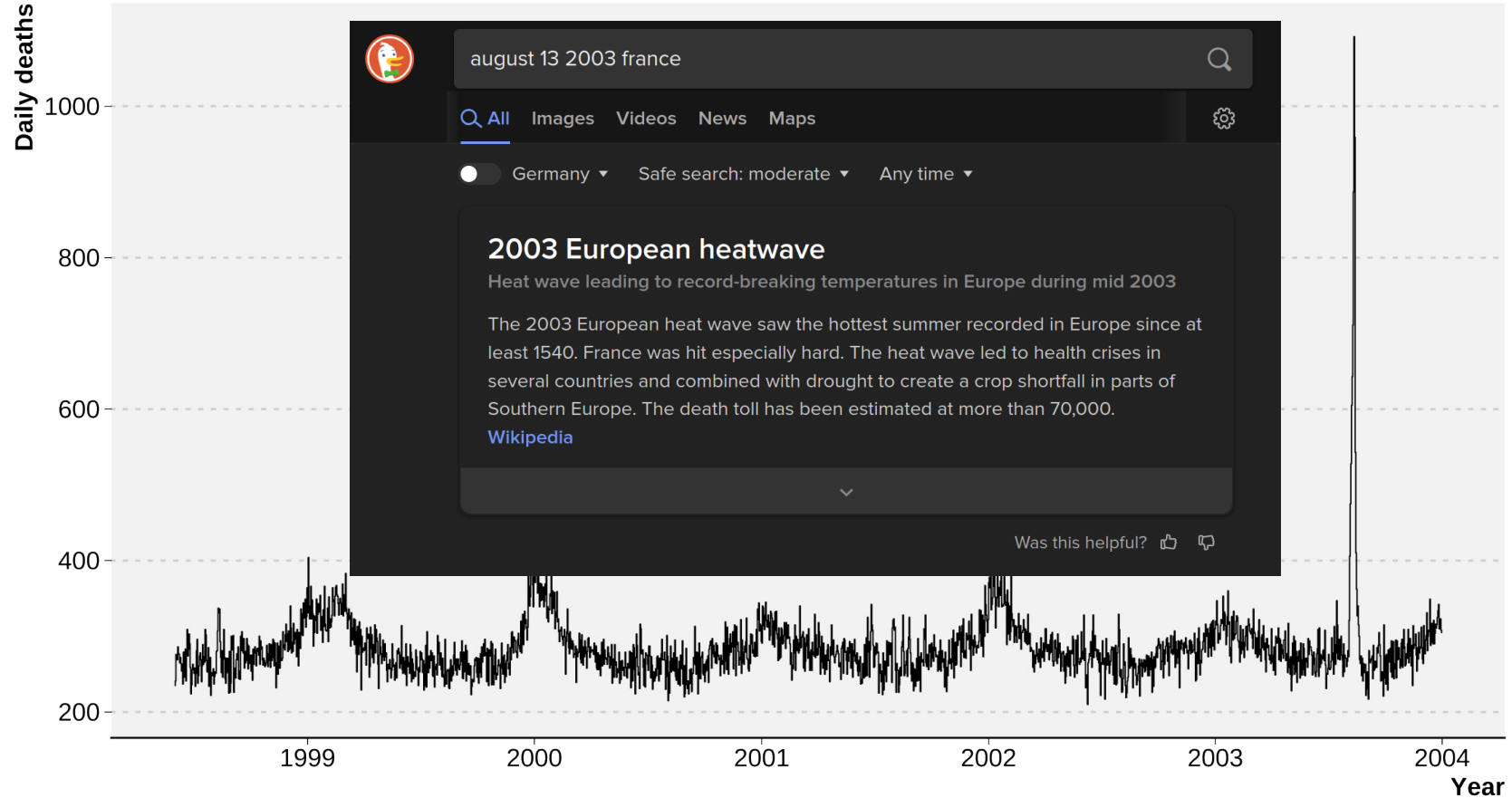
Mortality displacement during heatwaves



Data provided by Robert Chung

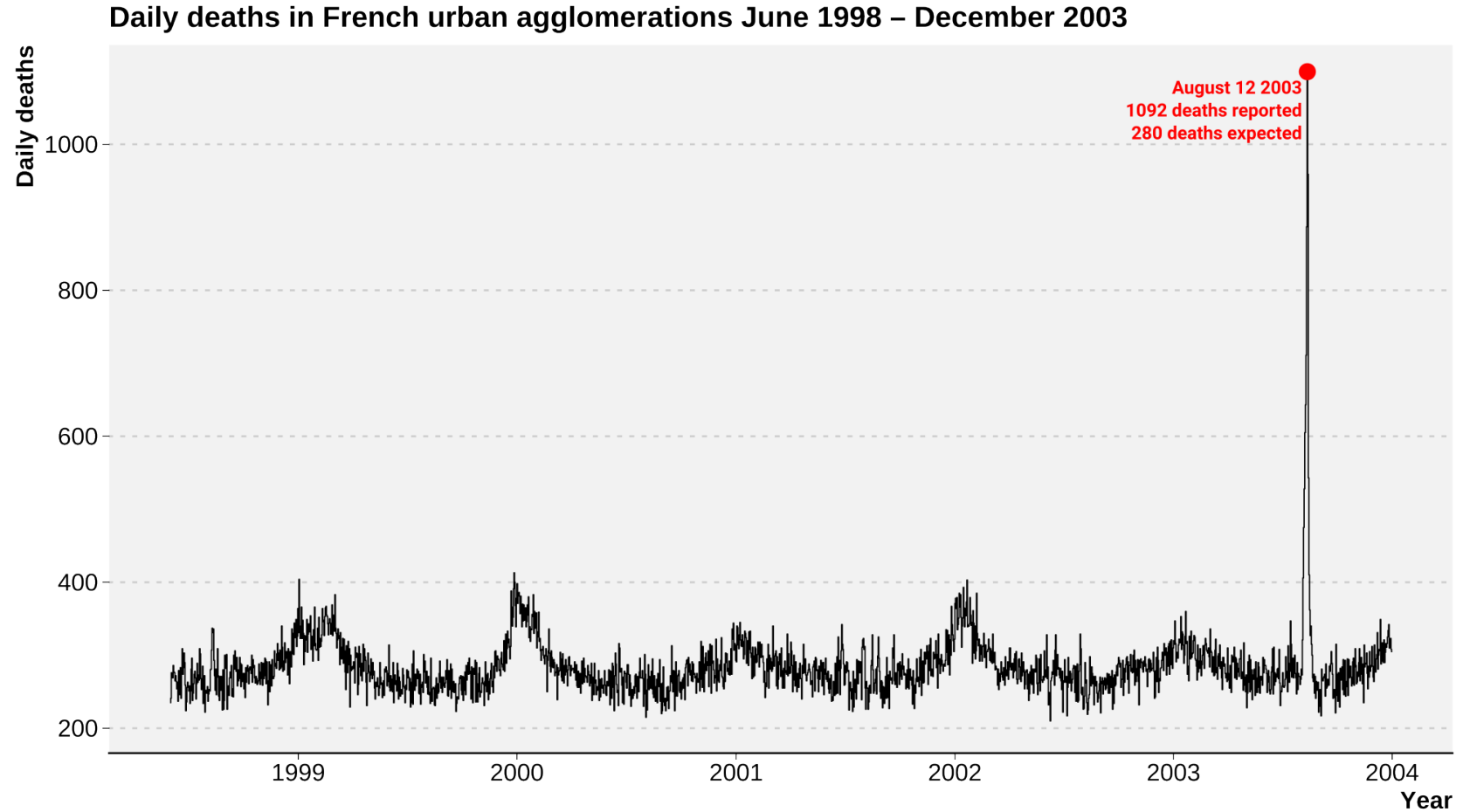
Mortality displacement during heatwaves

Daily deaths in French urban agglomerations June 1998 – December 2003



Data provided by Robert Chung

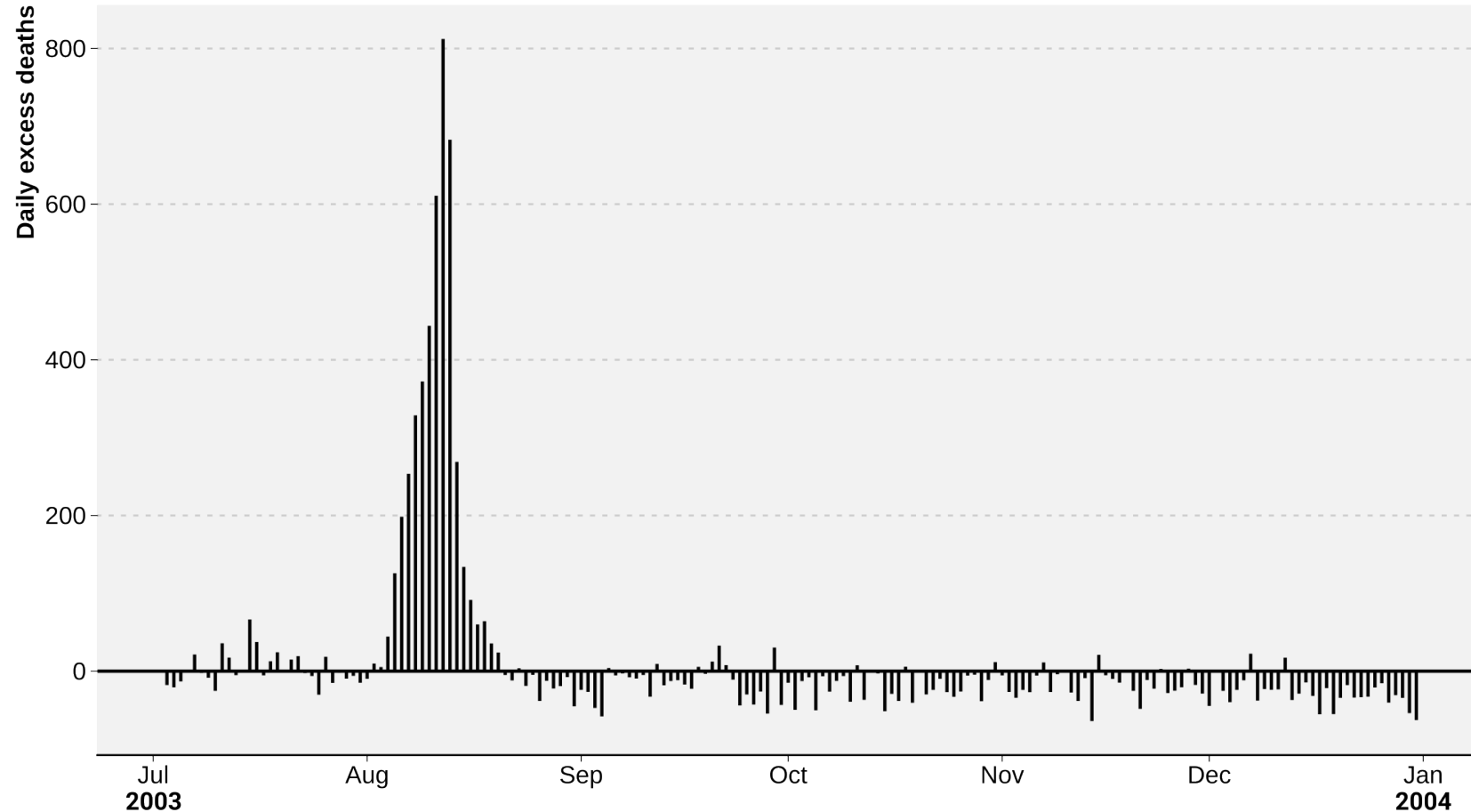
Mortality displacement during heatwaves



Data provided by Robert Chung

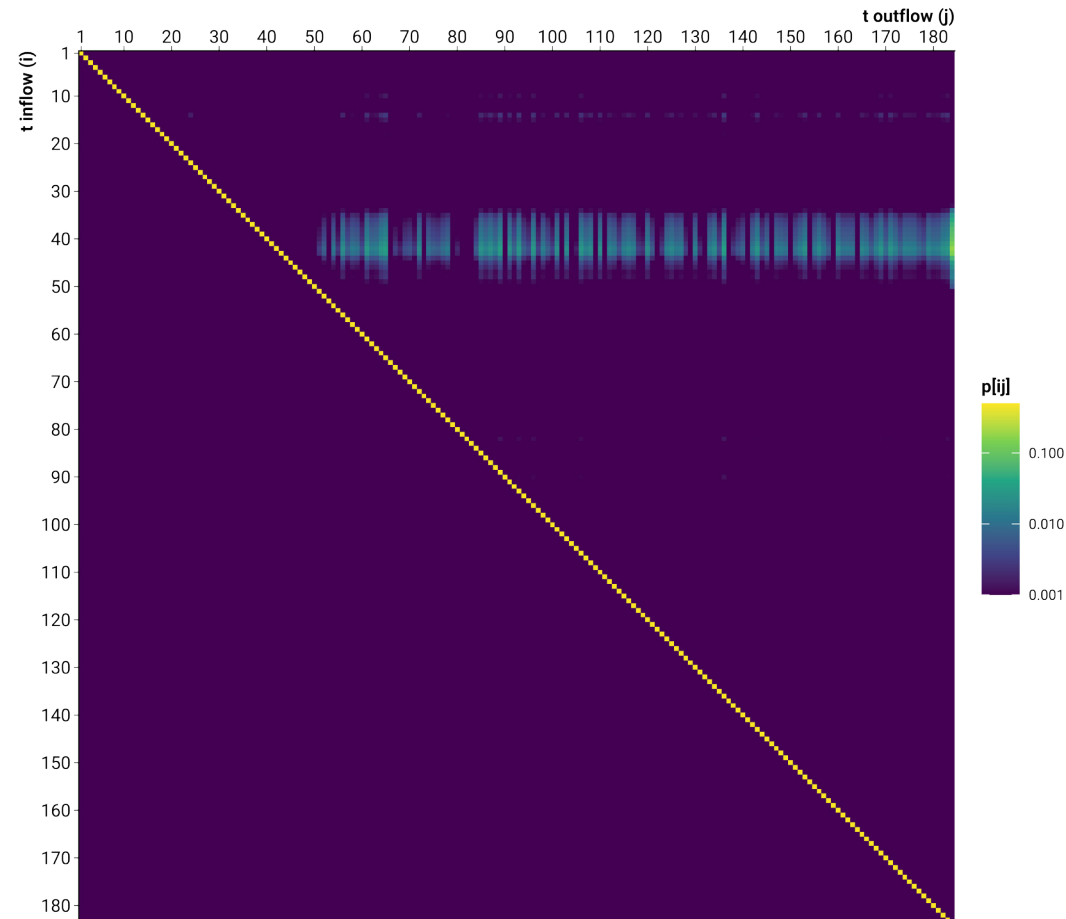
Mortality displacement during heatwaves

Daily excess deaths in French urban agglomerations July 2003 – Jan 2004

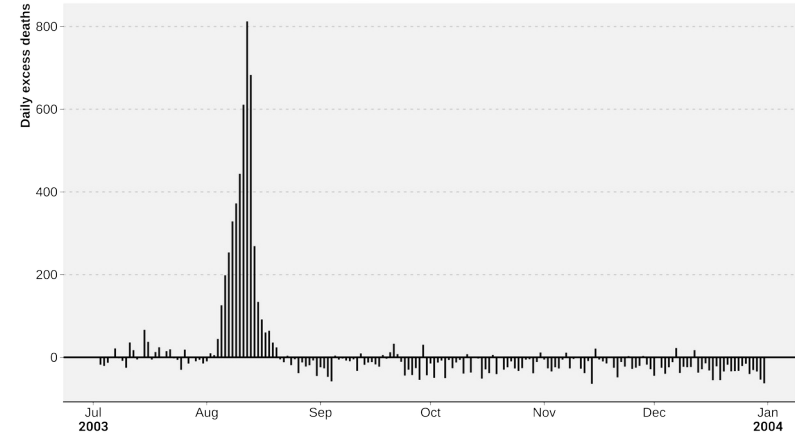


Mortality displacement during heatwaves

Displacement matrix under "all deficit to excess" estimation heuristic

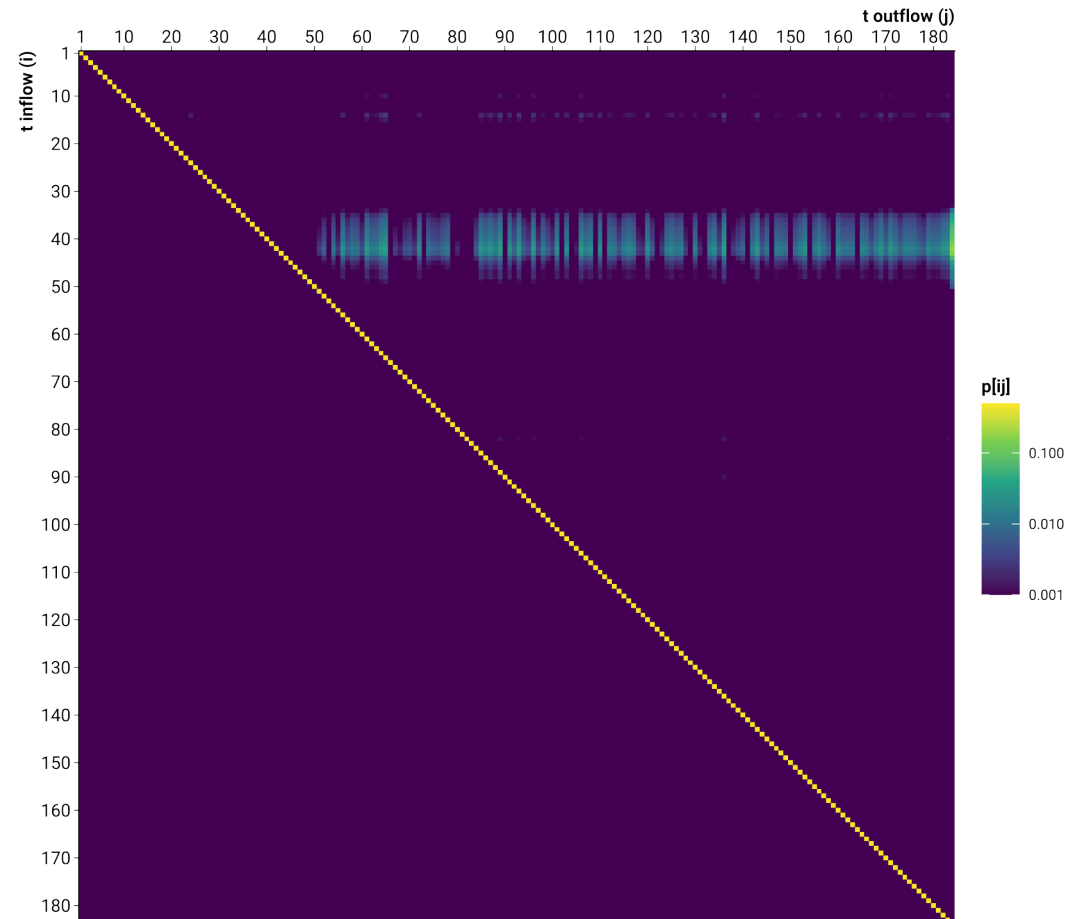


Daily excess deaths in French urban agglomerations July 2003 – Jan 2004

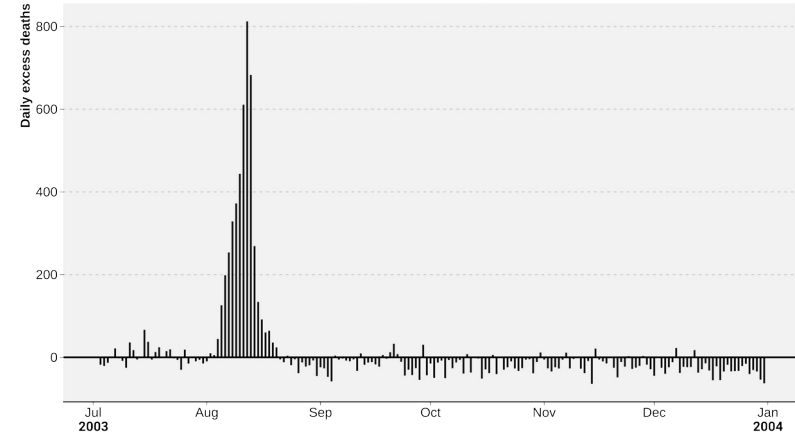


Mortality displacement during heatwaves

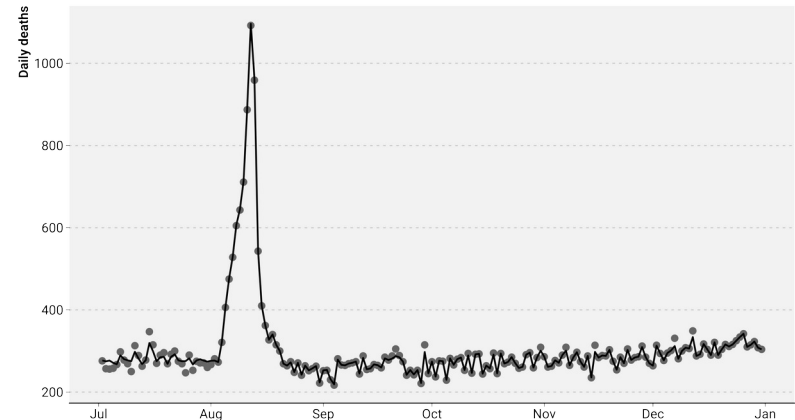
Displacement matrix under "all deficit to excess" estimation heuristic



Daily excess deaths in French urban agglomerations July 2003 – Jan 2004

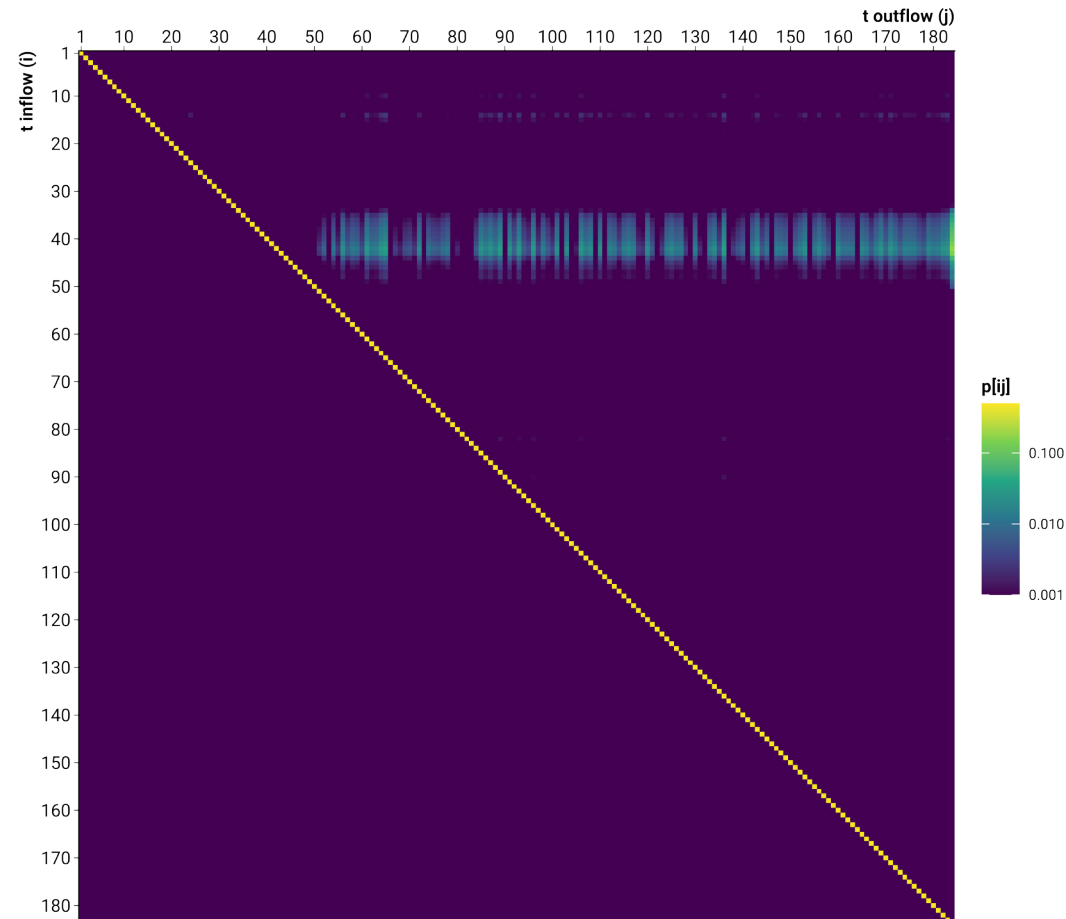


Daily observed and model predicted deaths in French urban agglomerations July 2003 – Jan 2004



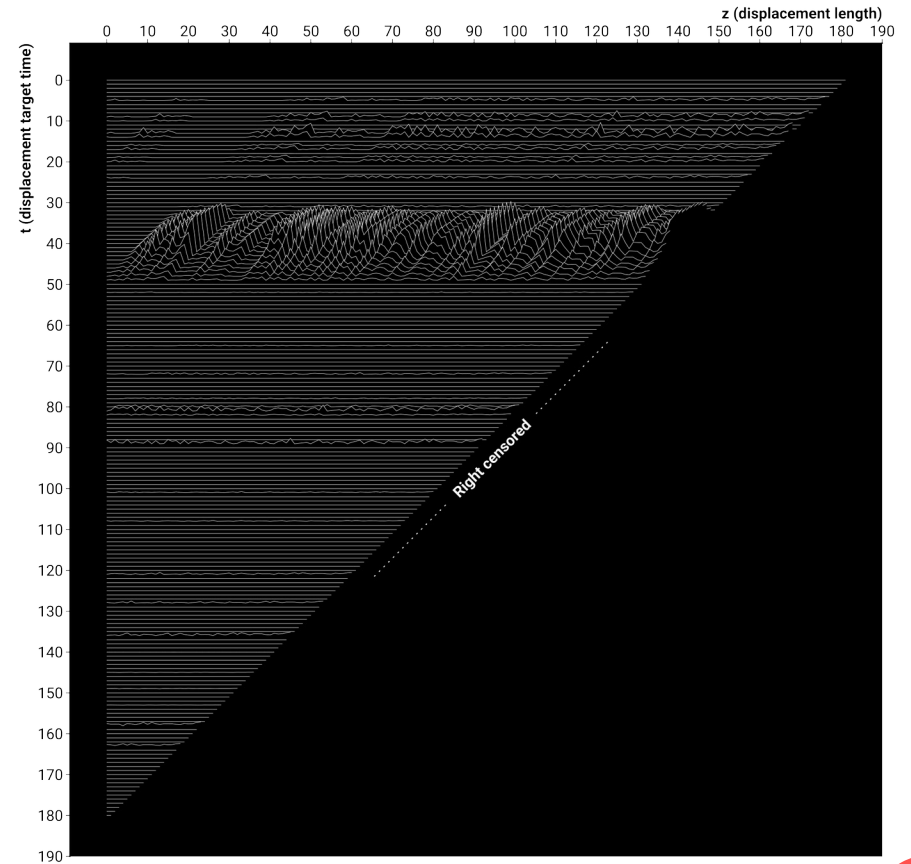
Mortality displacement during heatwaves

Displacement matrix under "all deficit to excess" estimation heuristic



Time dependent distribution of displacement lengths (days of life lost)

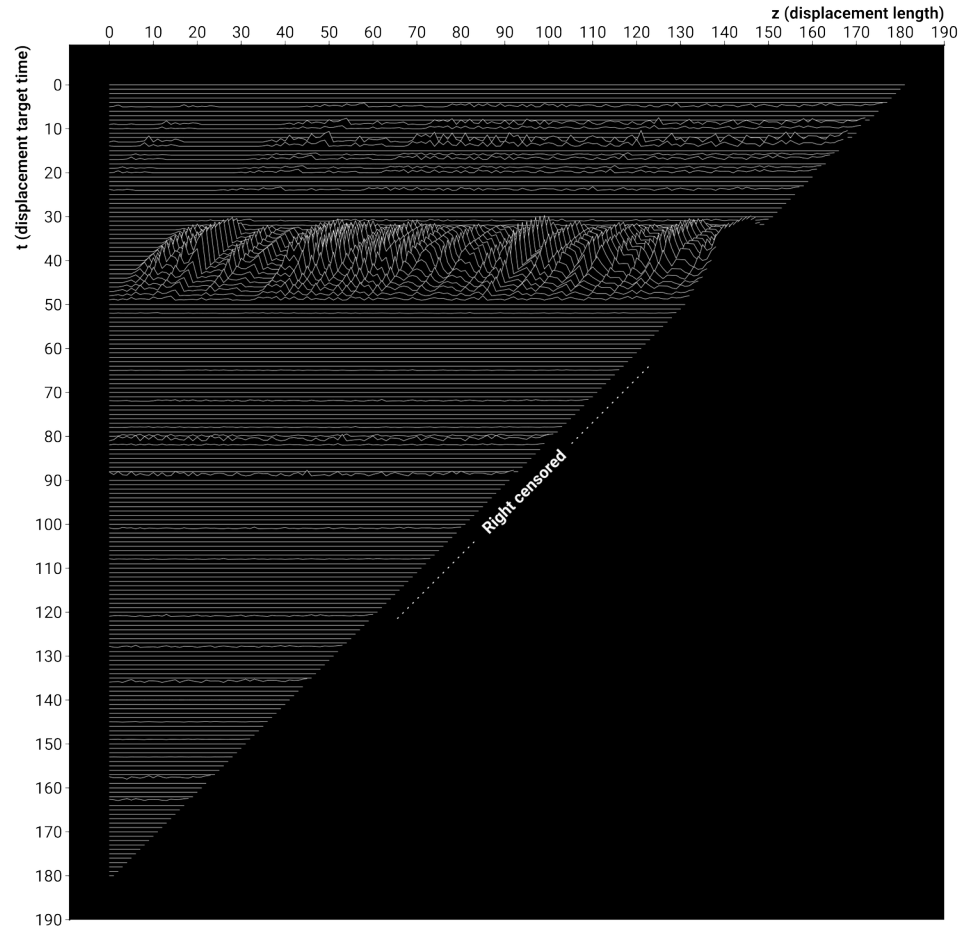
Right censoring and 0 displacement not shown.



Mortality displacement during heatwaves

Time dependent distribution of displacement lengths (days of life lost)

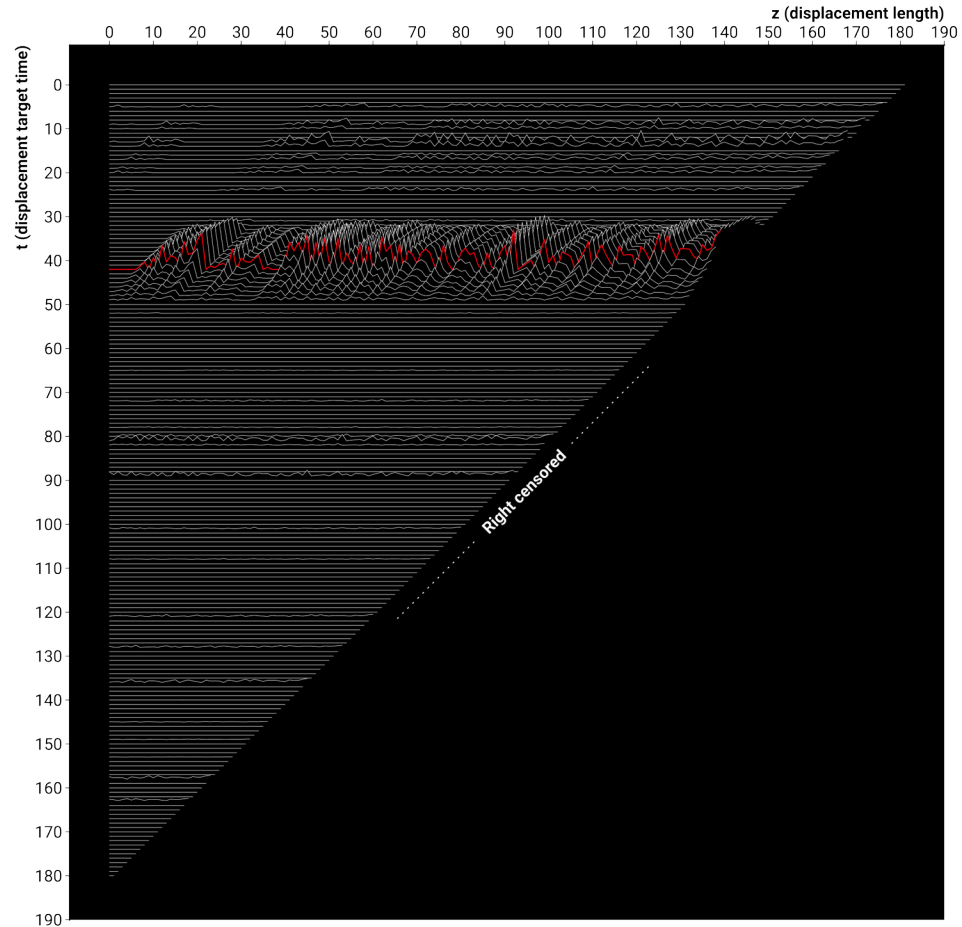
Right censoring and 0 displacement not shown.



Mortality displacement during heatwaves

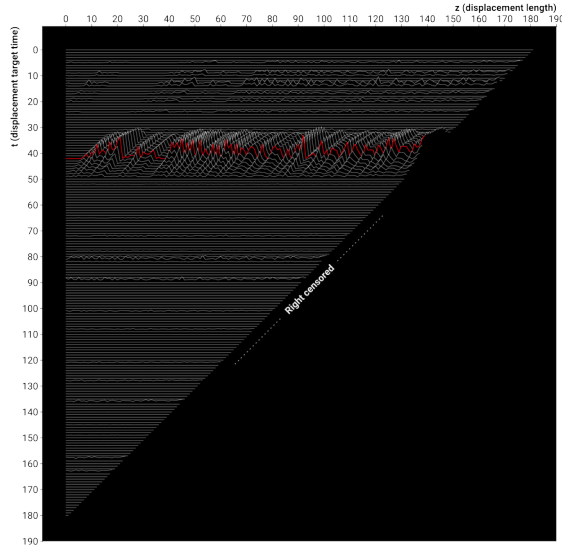
Time dependent distribution of displacement lengths (days of life lost)

Right censoring and 0 displacement not shown.

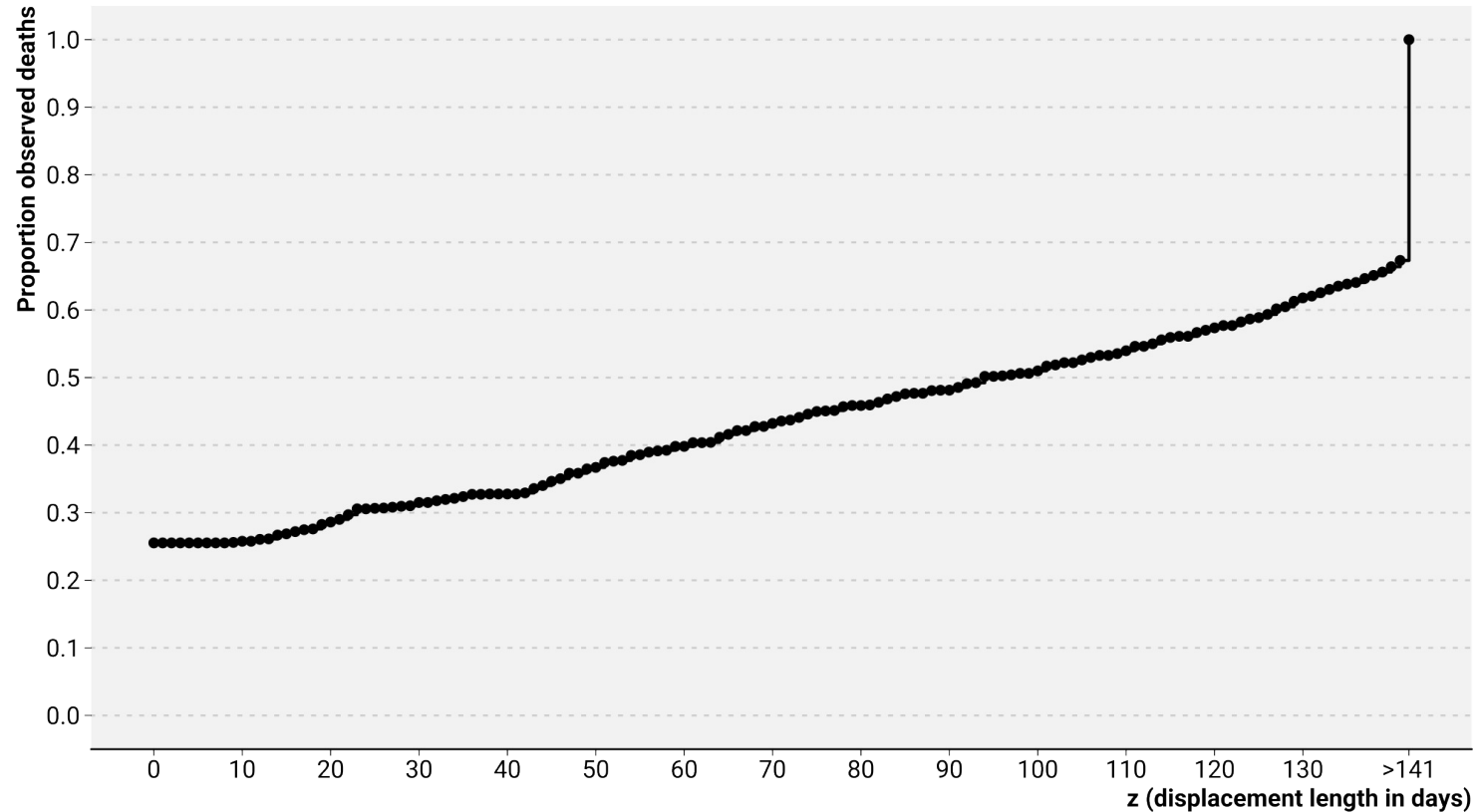


Mortality displacement during heatwaves

Time dependent distribution of displacement lengths (days of life lost)
Right censoring and 0 displacement not shown.

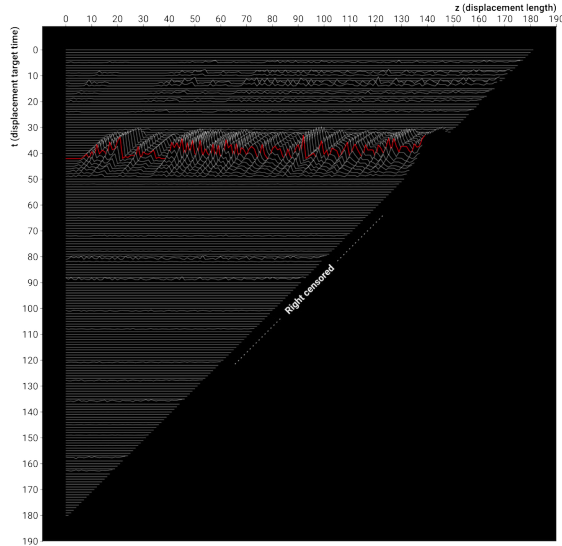


Proportion deaths at t=42 by displacement length

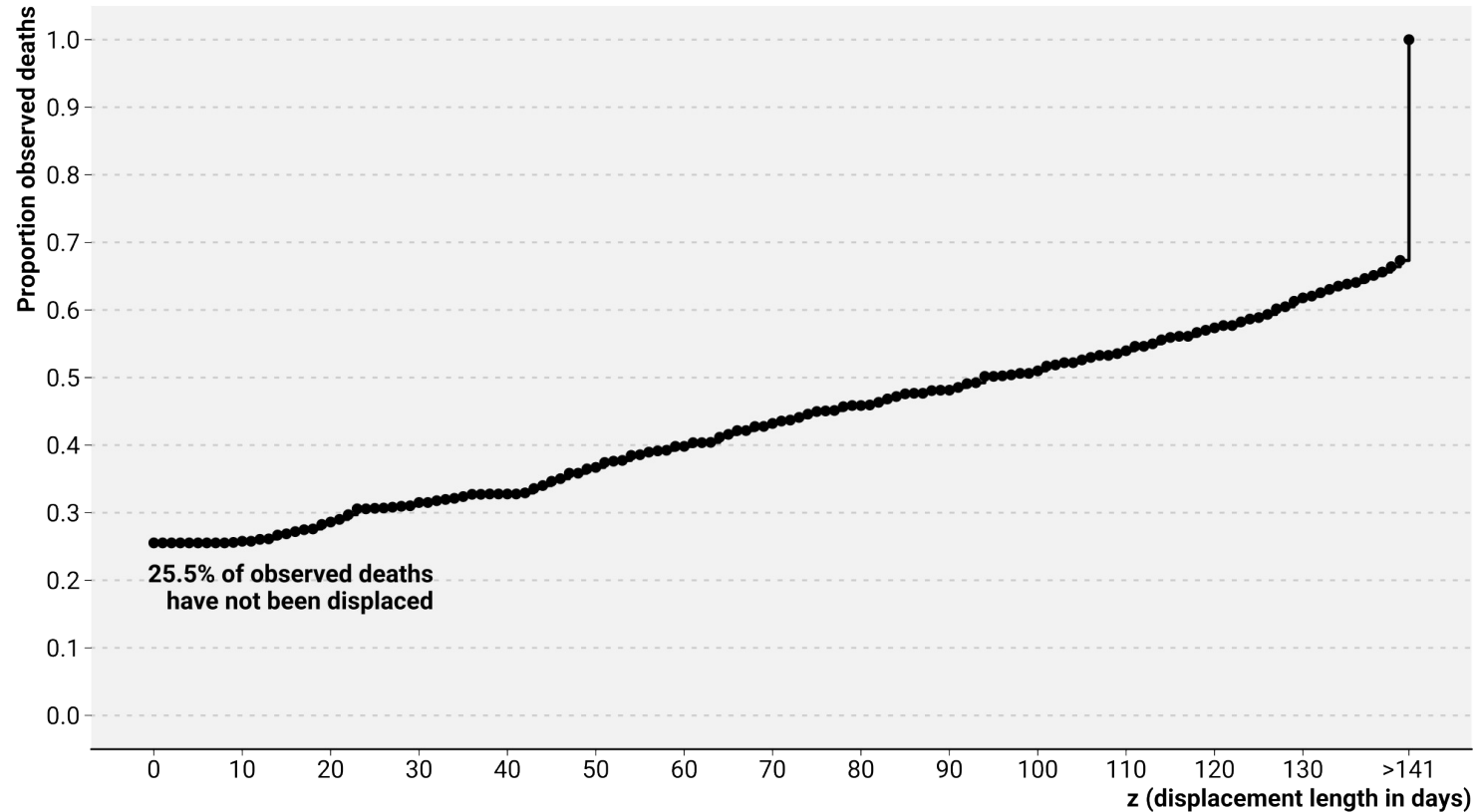


Mortality displacement during heatwaves

Time dependent distribution of displacement lengths (days of life lost)
Right censoring and 0 displacement not shown.

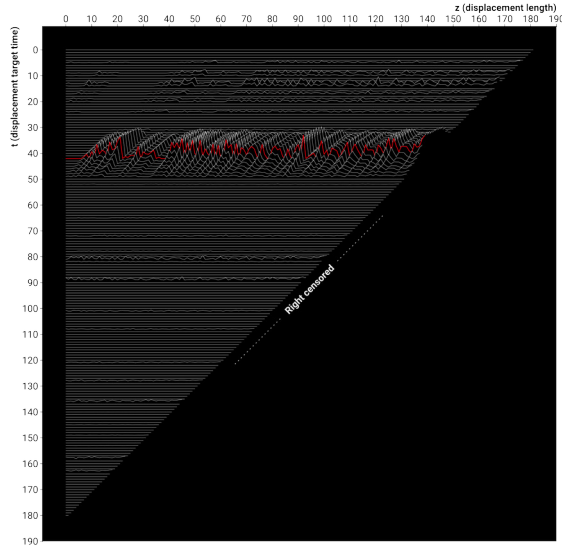


Proportion deaths at t=42 by displacement length

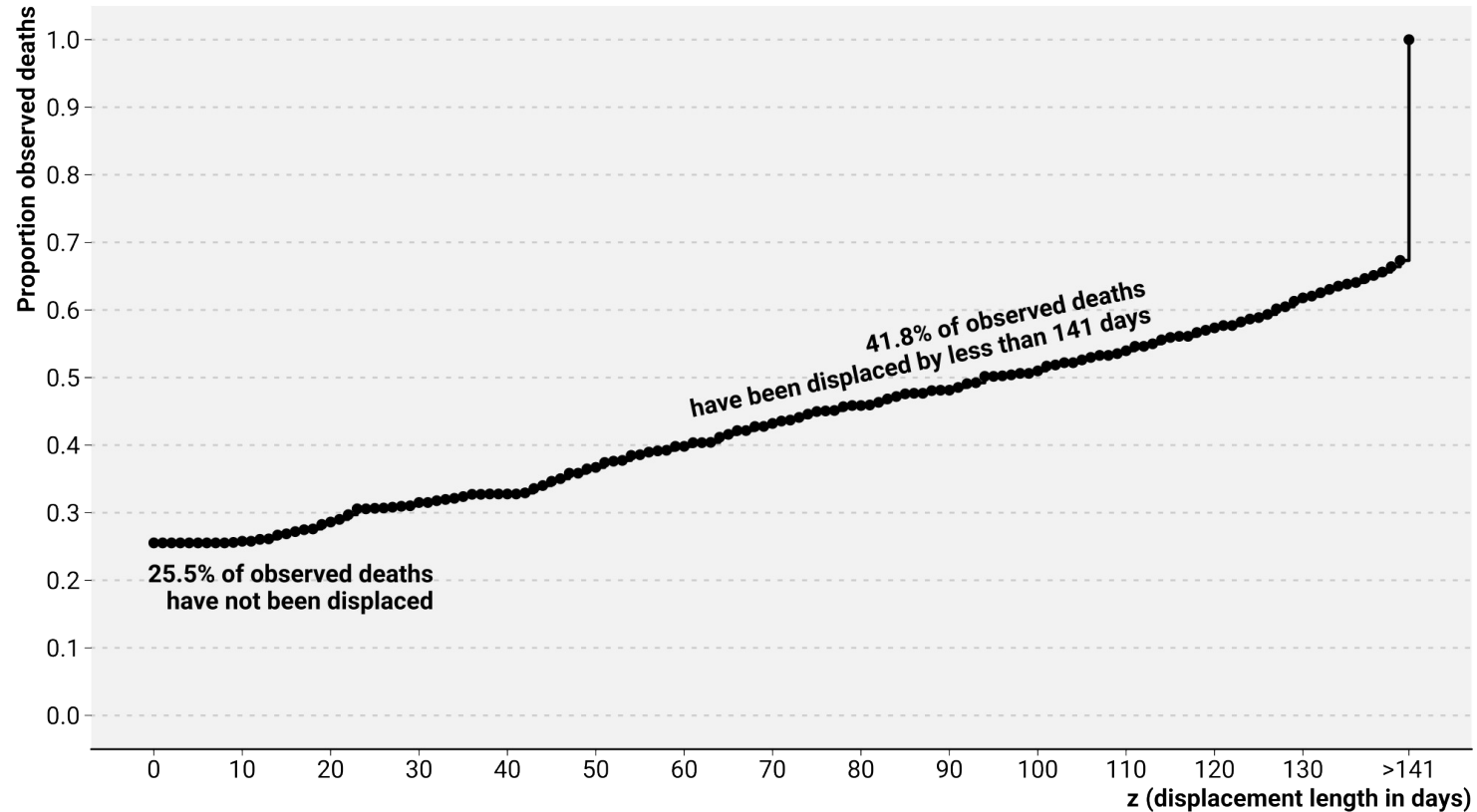


Mortality displacement during heatwaves

Time dependent distribution of displacement lengths (days of life lost)
Right censoring and 0 displacement not shown.

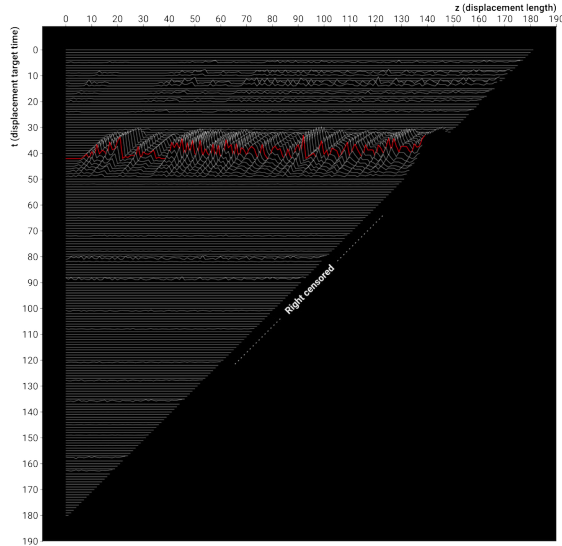


Proportion deaths at t=42 by displacement length

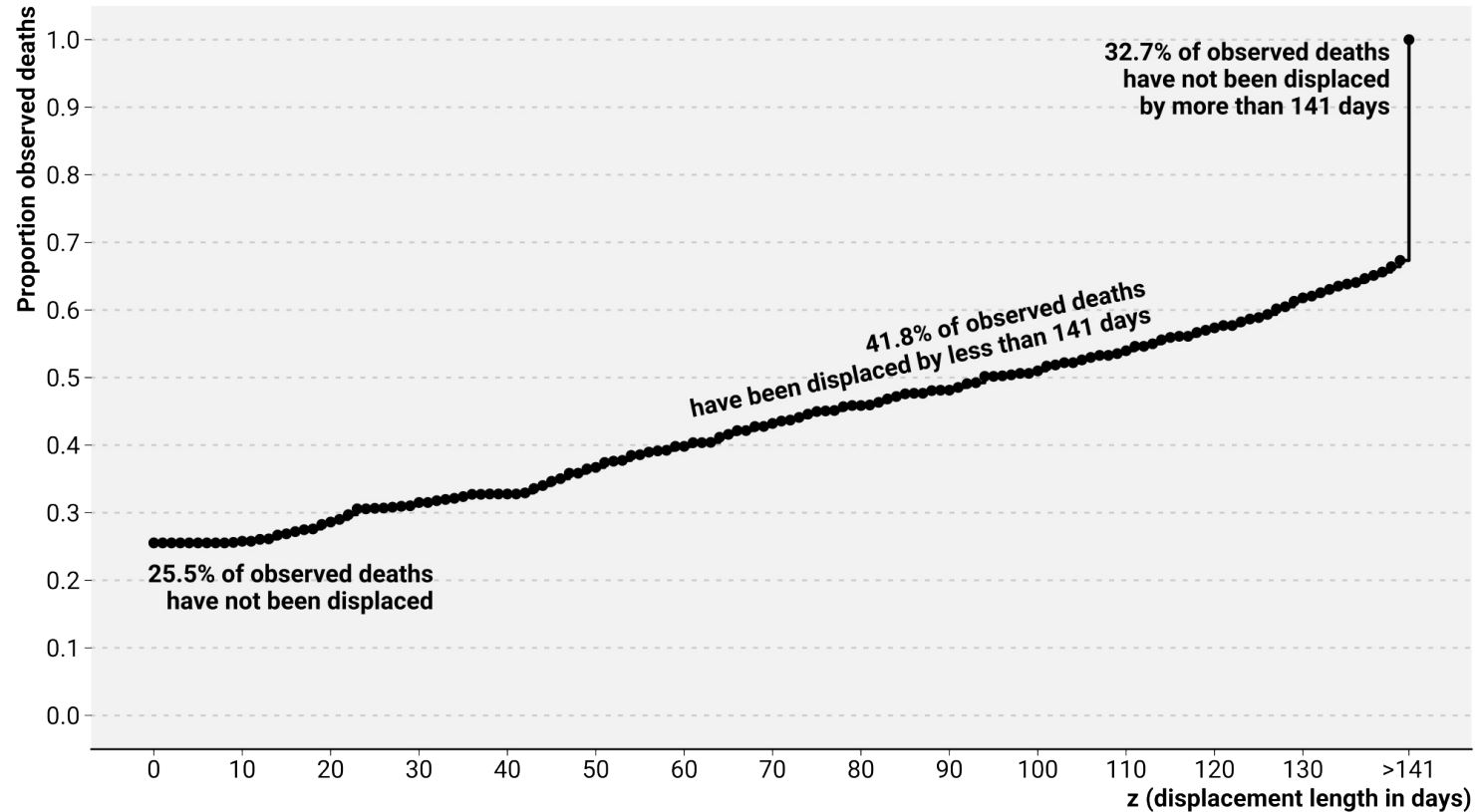


Mortality displacement during heatwaves

Time dependent distribution of displacement lengths (days of life lost)
Right censoring and 0 displacement not shown.



Proportion deaths at $t=42$ by displacement length



Convolution in survival analysis

Convolution in survival analysis

$$f_Y(y) = \int_0^{\omega} f_X(x = y+z) f_Z(z) dz$$

$$Y = X - Z$$

Actual age at death = Age at death under expected scenario – Lifetime lost

Convolution in survival analysis

Independent displacement

$$f_Y(y) = \int_0^{\omega} f_X(x = y+z) f_Z(z) dz$$

$$Y = X - Z$$

Actual age at death = Age at death under expected scenario – Lifetime lost

Age dependent displacement

$$f_Y(y) = \int_0^{\omega} \int_0^{\omega} f_X(x = y+z) f_{Z|X}(z|x) dz dx$$

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