Lifetime lost from time series of counts

A convolution formulation of mortality displacement

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Everything in mortality is a tempo effect

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Displaced deaths

But they may only have lived for another month anyway...

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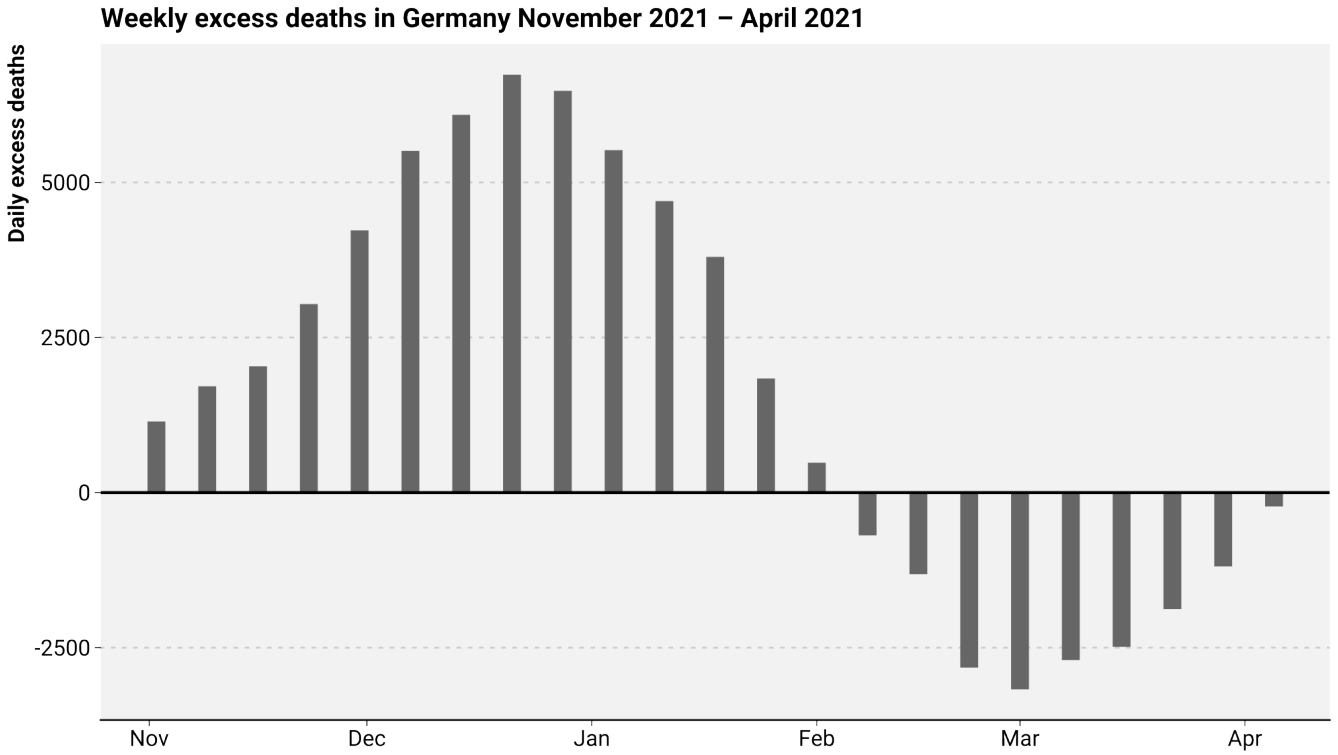


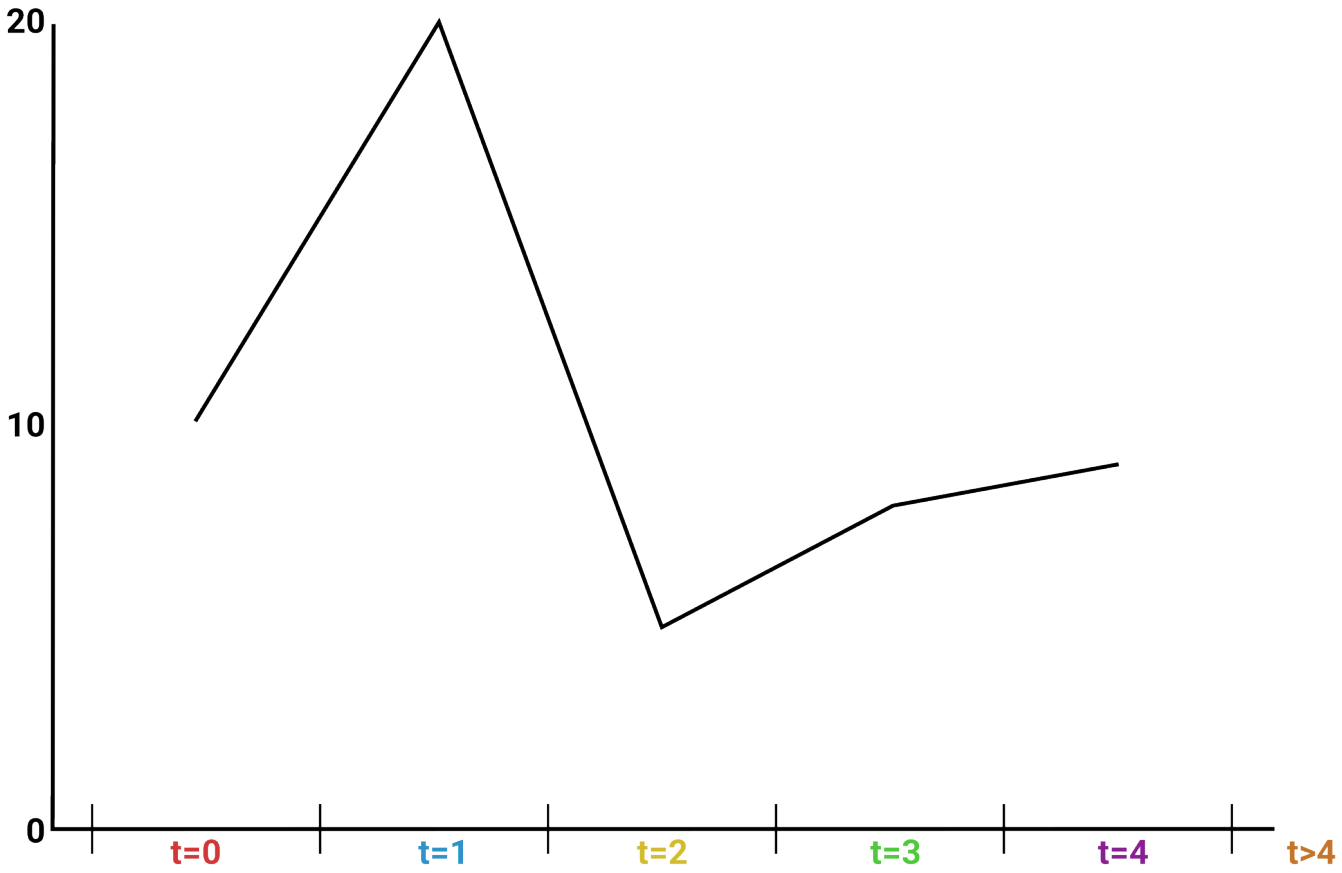
Displaced deaths

Observed deaths are expected deaths displaced in time

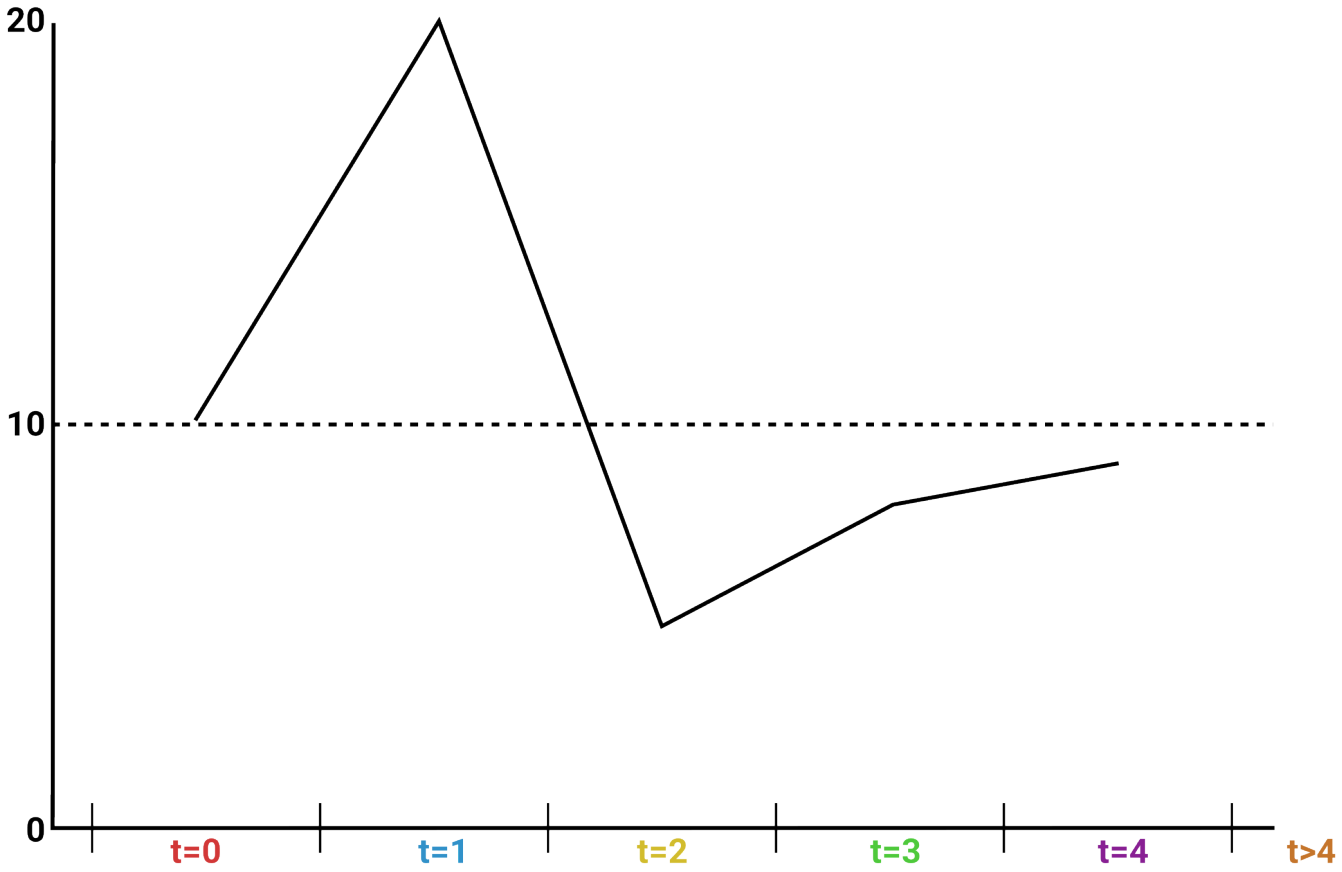
aka

In mortality there are only tempo effects

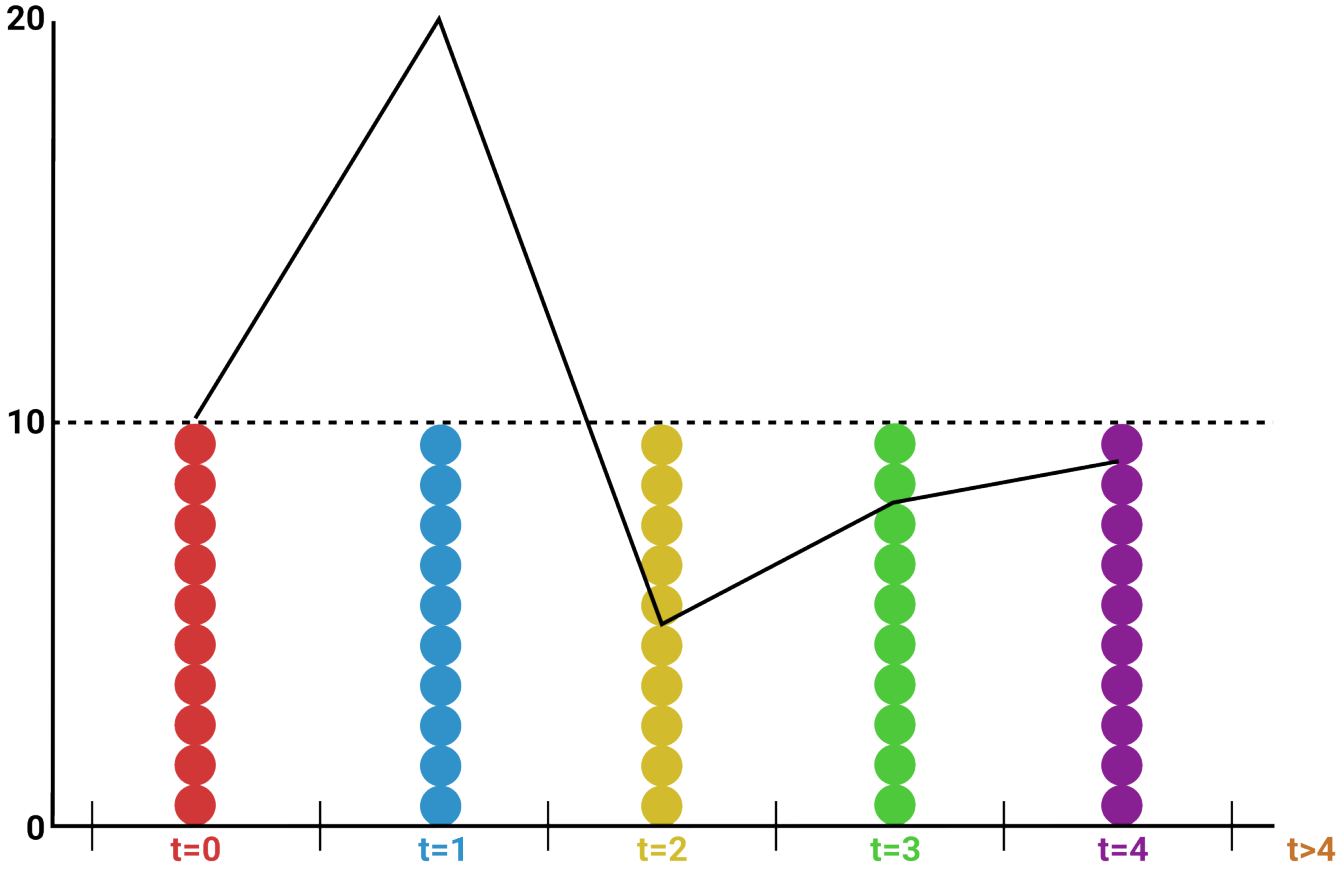
Displaced deaths



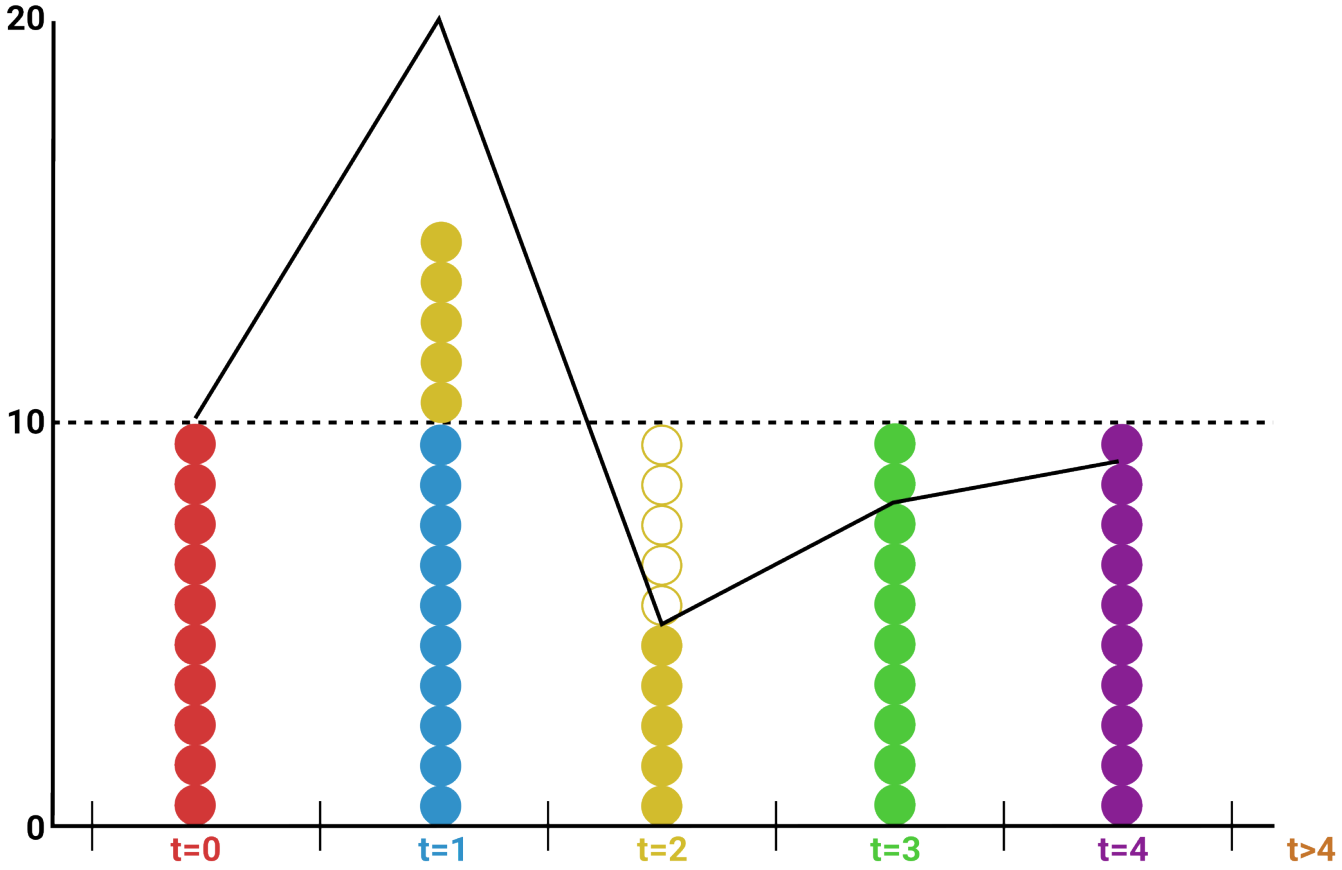
Displaced deaths



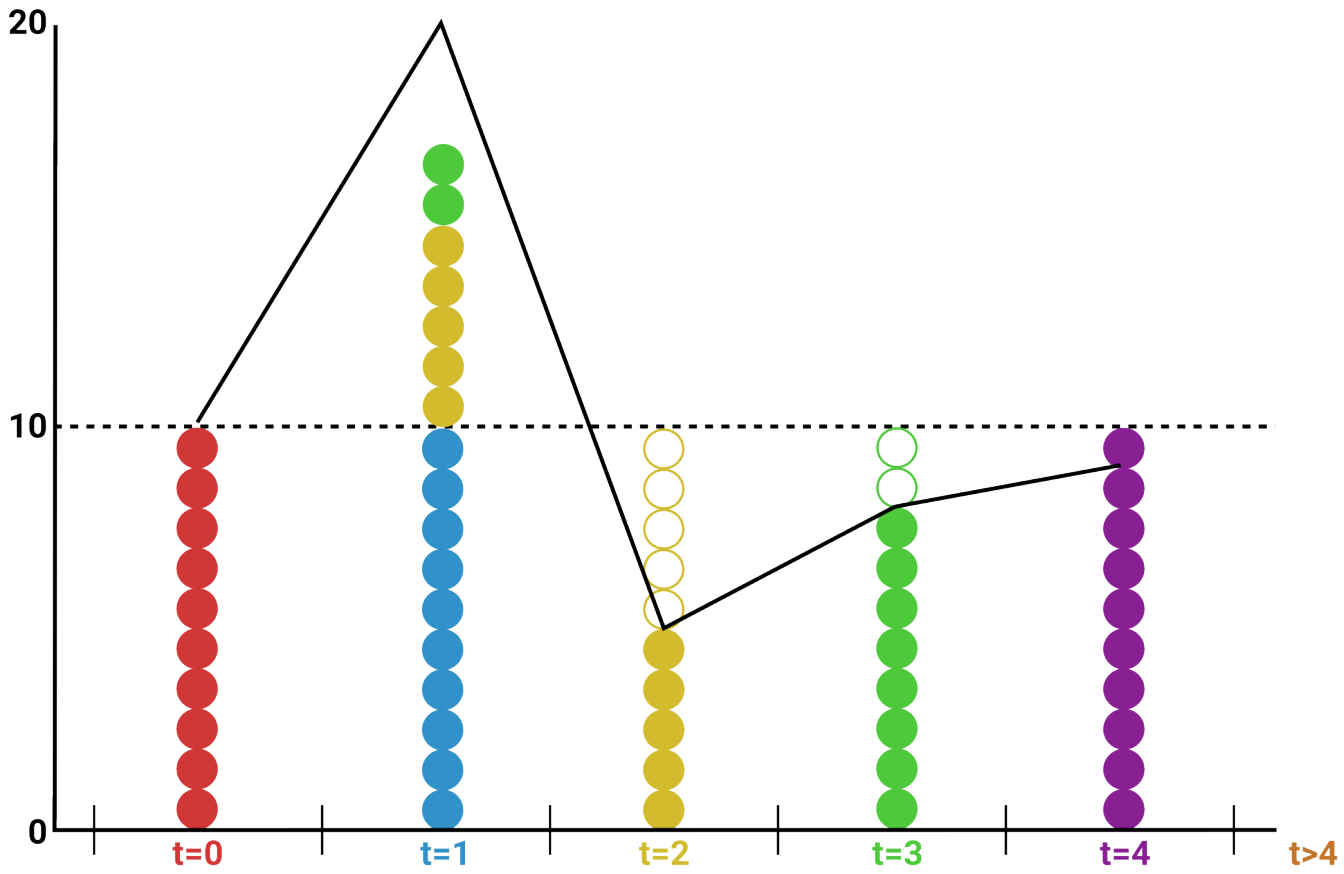
Displaced deaths



Displaced deaths



Displaced deaths



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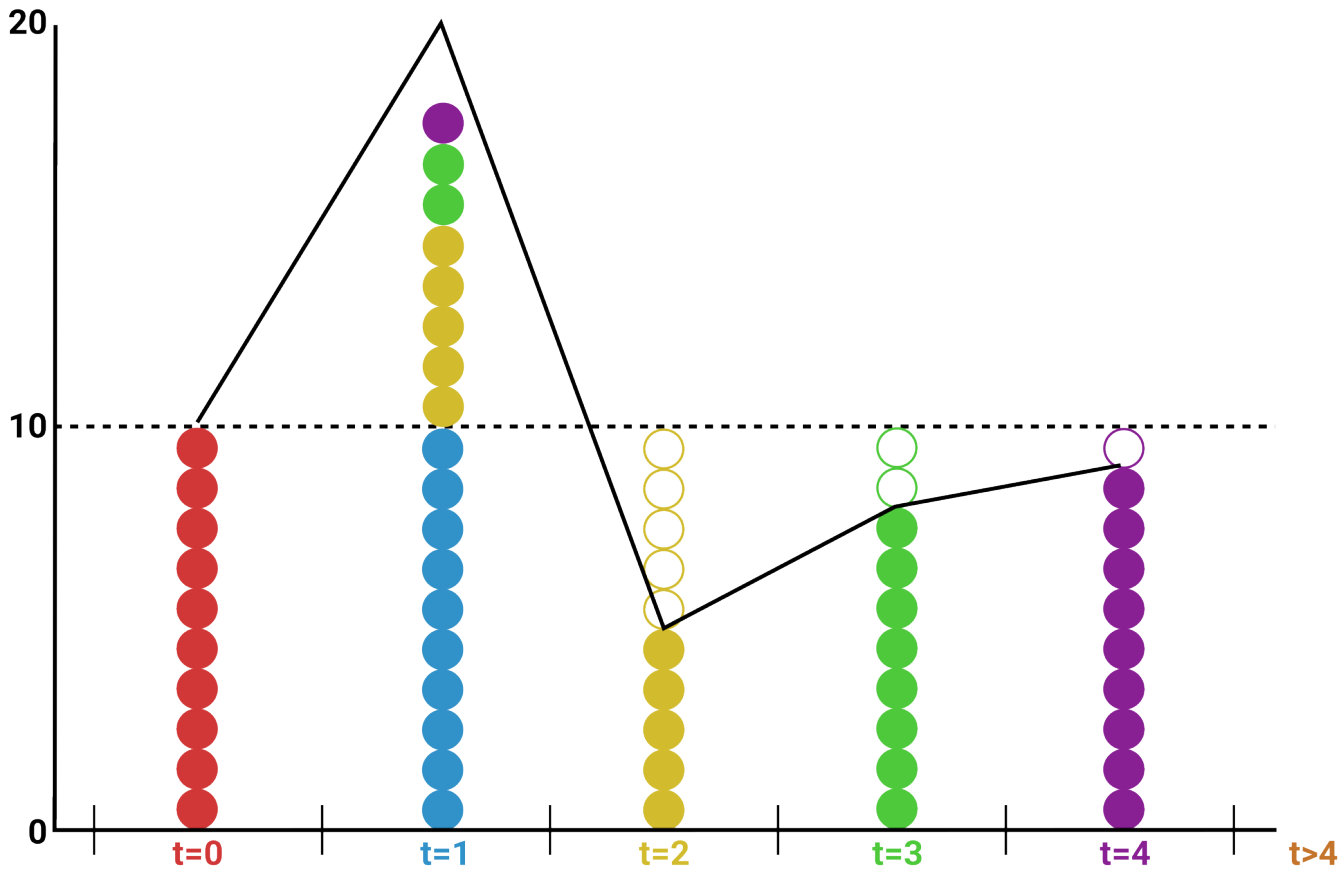
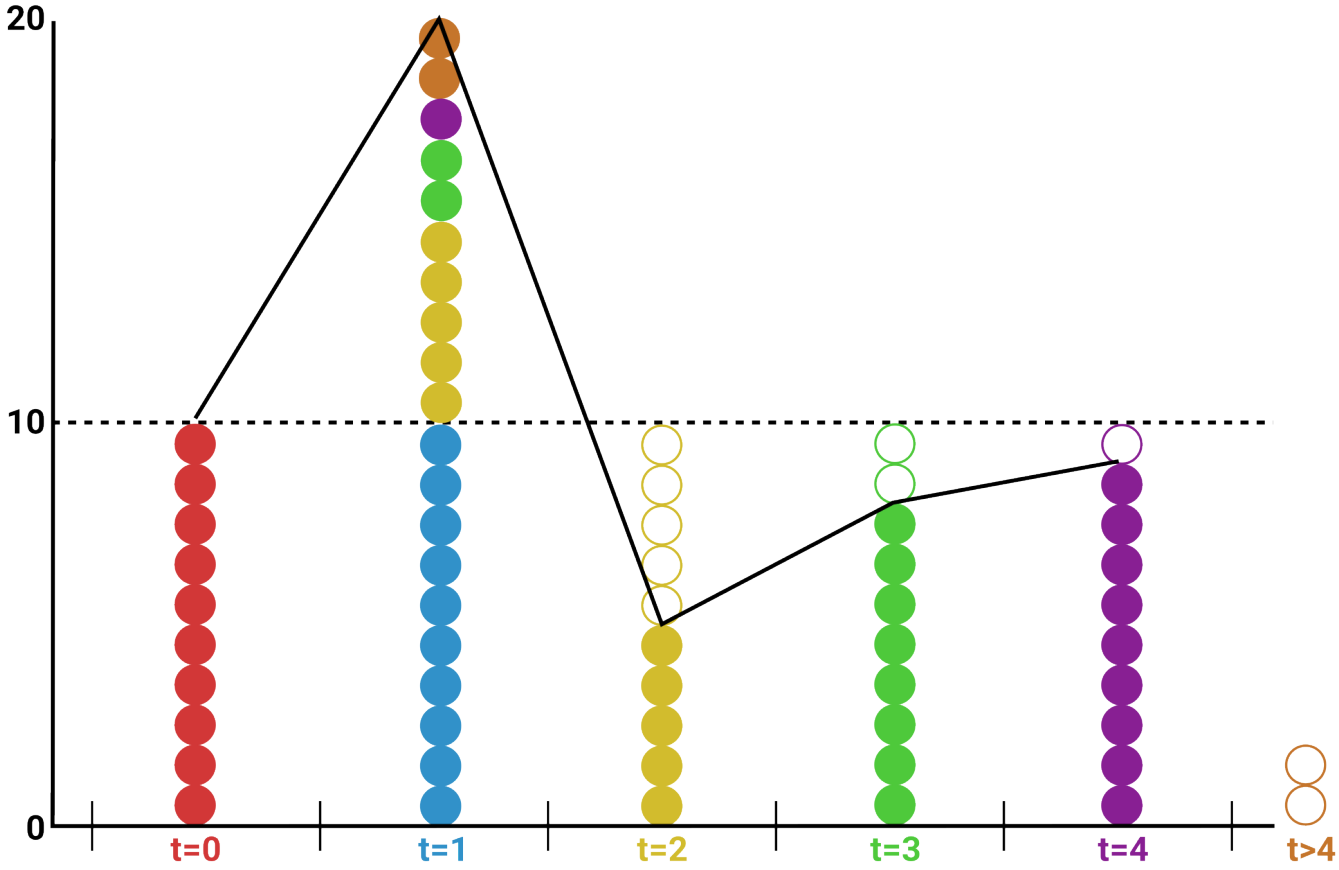
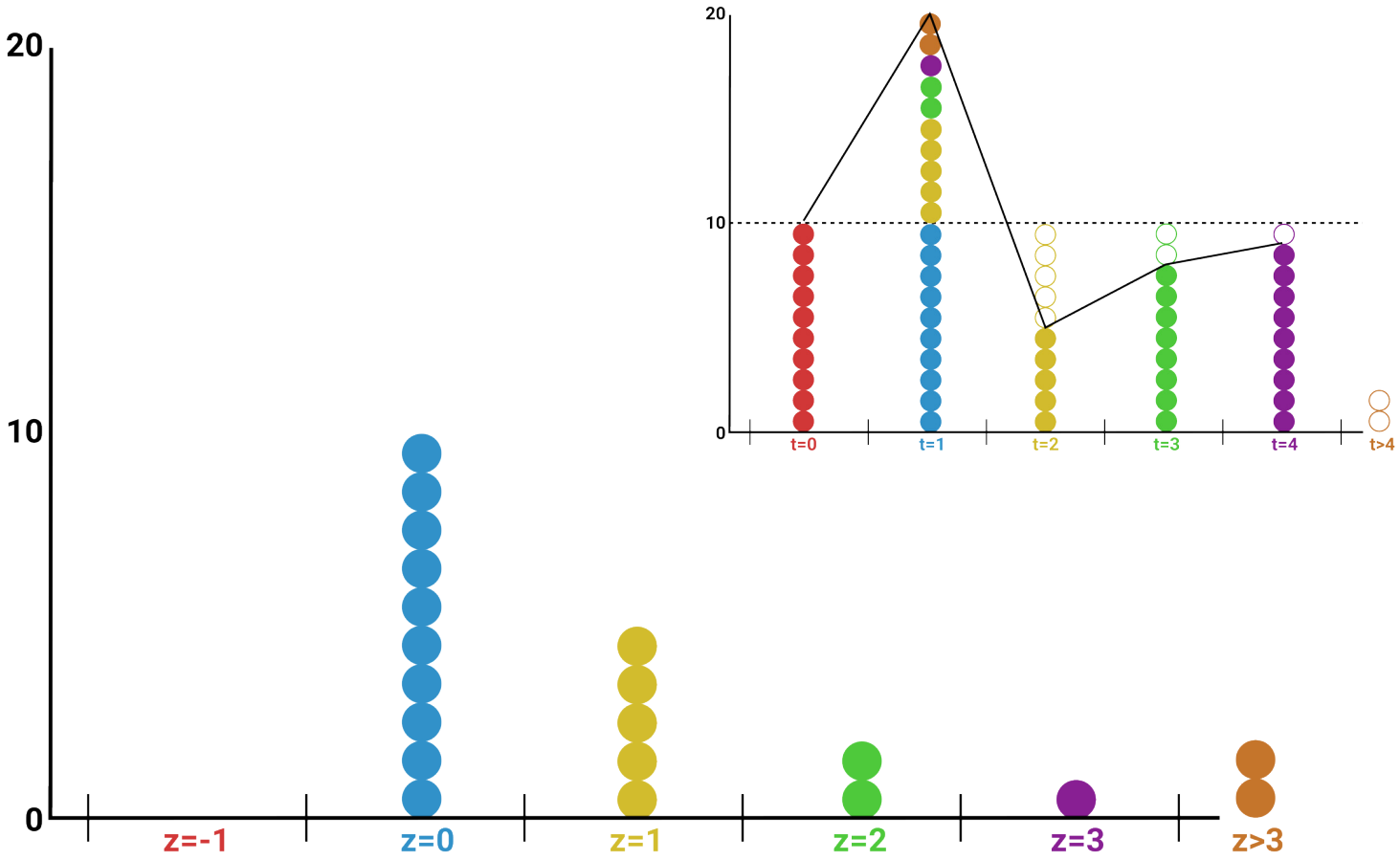


Displaced deaths



Displaced deaths



The displacement matrix

Displaced deaths

Observed deaths are expected deaths displaced in time

$$y = Ax$$

The displacement matrix

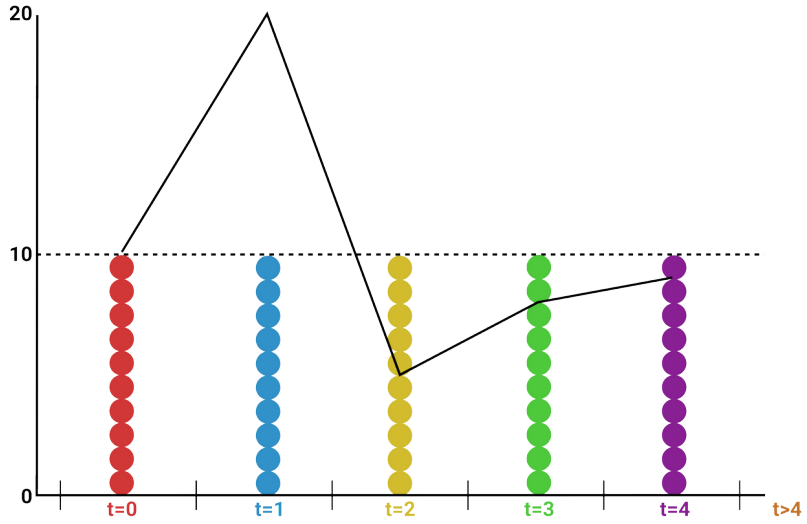
$$p = N^2 + N$$

y=Ax

Expectation vector **x**

10	10	10	10	10	$\sum_{t=0:4} y_t - x_t$
					2

Observation vector y						
10	p_{12}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
20	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}
5	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}
8	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}
9	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}



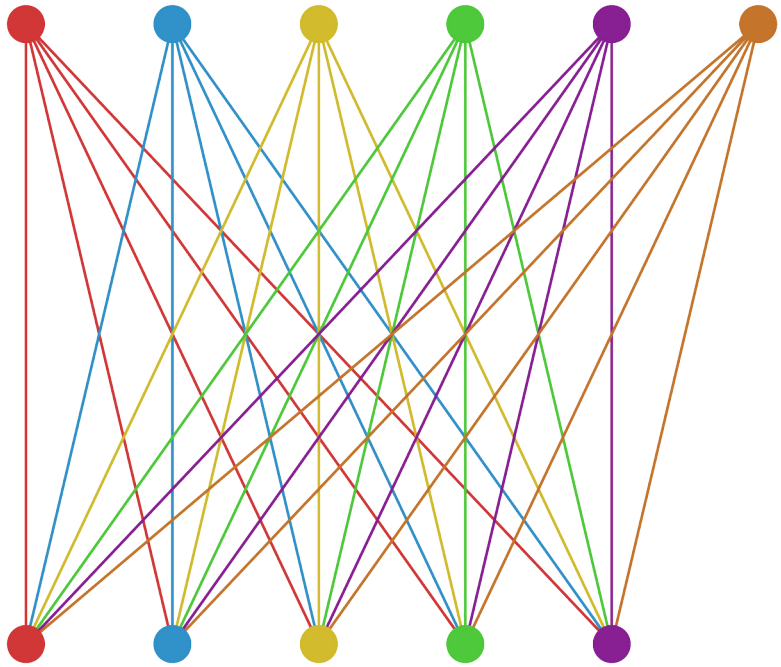
The displacement matrix

$$y = Ax$$

$$\sum_{t=0:4} y_t - x_t$$

	10	10	10	10	10	2
10	p_{12}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
20	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}
5	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}
8	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}
9	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}

$$p = N^2 + N$$

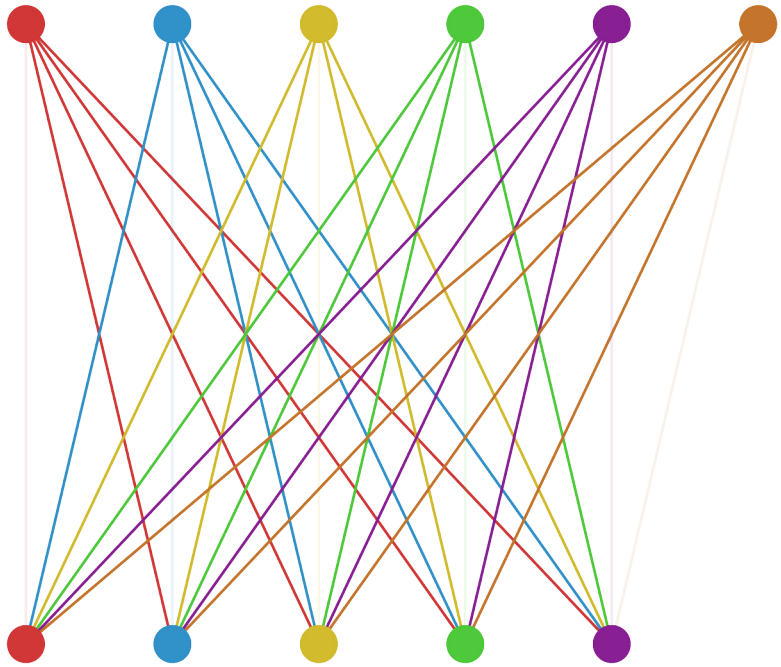


The displacement matrix

$$y = Ax$$

$$\begin{array}{c|cccccc|c} & 10 & 10 & 10 & 10 & 10 & 2 & \sum_{t=0:4} y_t - x_t \\ \hline 10 & 1-\sum_i p_{i1} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} & \\ 20 & p_{21} & 1-\sum_i p_{i2} & p_{23} & p_{24} & p_{25} & p_{26} & \\ 5 & p_{31} & p_{32} & 1-\sum_i p_{i3} & p_{34} & p_{35} & p_{36} & \\ 8 & p_{41} & p_{42} & p_{43} & 1-\sum_i p_{i4} & p_{45} & p_{46} & \\ 9 & p_{51} & p_{52} & p_{53} & p_{54} & 1-\sum_i p_{i5} & 1-\sum_i p_{i6} & \end{array}$$

$$p = N^2 - 1$$



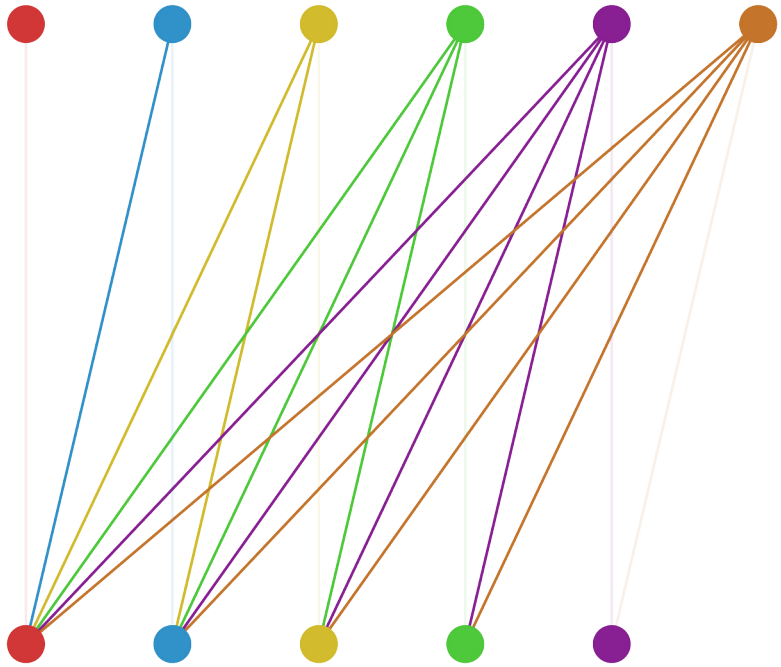
Pure displacement

The displacement matrix

$$p = 1/2(N + 2)(N - 1)$$

$y = Ax$

	10	10	10	10	10	$\sum_{t=0:4} y_t - x_t$
10	$1-\sum_i p_{i1}$	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
20	0	$1-\sum_i p_{i2}$	p_{23}	p_{24}	p_{25}	p_{26}
5	0	0	$1-\sum_i p_{i3}$	p_{34}	p_{35}	p_{36}
8	0	0	0	$1-\sum_i p_{i4}$	p_{45}	p_{46}
9	0	0	0	0	$1-\sum_i p_{i5}$	$1-\sum_i p_{i6}$



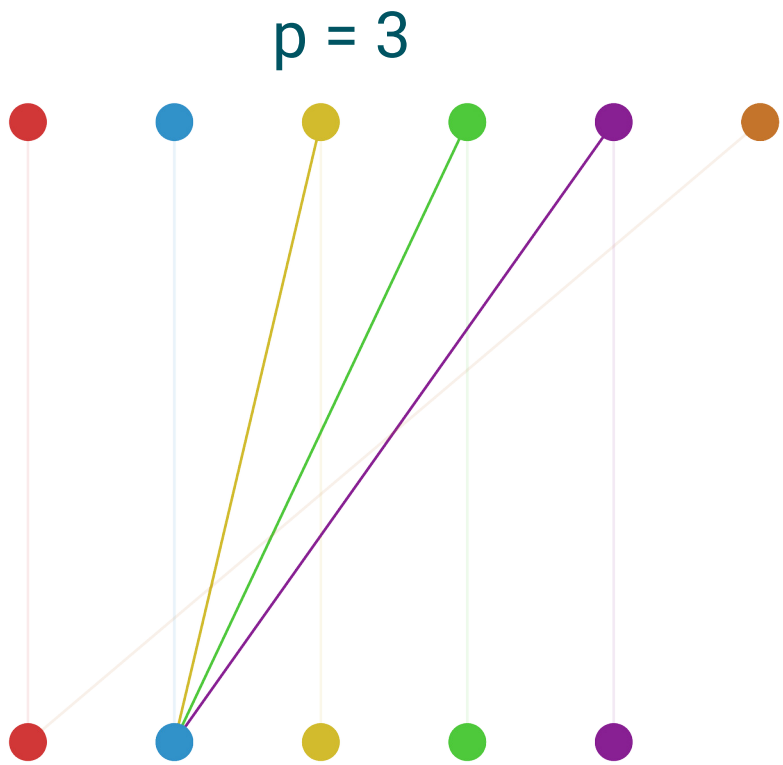
Pure displacement + No lifesaving

The displacement matrix

$$y = Ax$$

$$\sum_{t=0:4} y_t - x_t$$

	10	10	10	10	10	2
10	$1-\sum_i p_{i1}$	0	0	0	0	0
20	0	$1-\sum_i p_{i2}$	p_{23}	p_{24}	p_{25}	1
5	0	0	$1-\sum_i p_{i3}$	0	0	0
8	0	0	0	$1-\sum_i p_{i4}$	p_{45}	0
9	0	0	0	0	$1-\sum_i p_{i5}$	0



Pure displacement + No lifesaving + Life lost only during excess

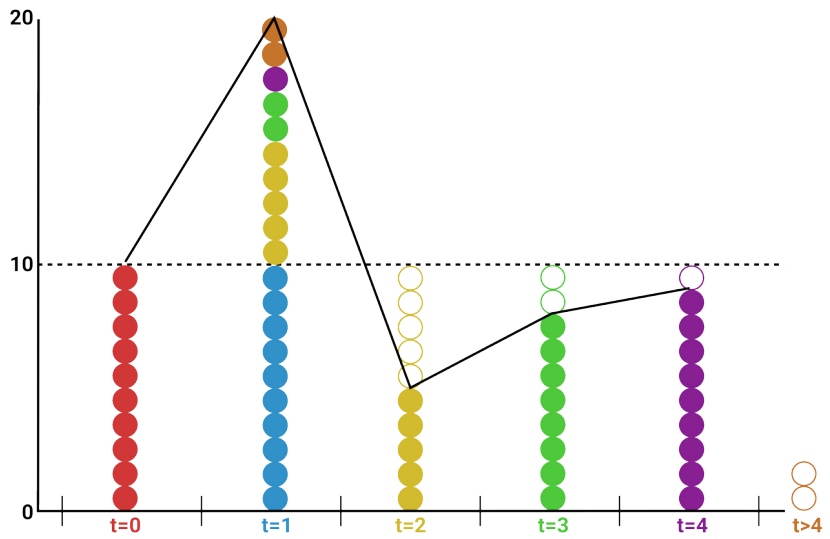
The displacement matrix

p = 3

$\sum_{t=0:4} y_t - x_t$

y=Ax

	10	10	10	10	10	2
10	1	0	0	0	0	0
20	0	1	.5	.2	.1	1
5	0	0	.5	0	0	0
8	0	0	0	.8	0	0
9	0	0	0	0	.9	0

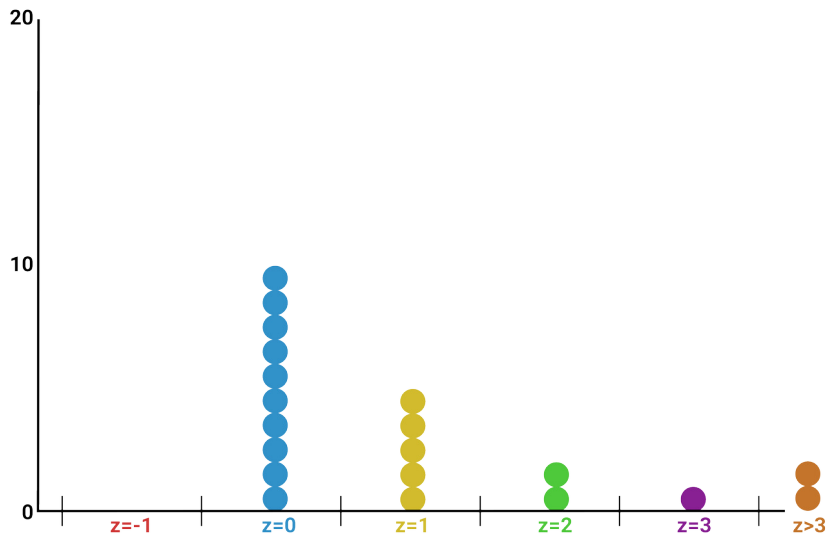


Pure displacement + No lifesaving + Life lost only during excess

The displacement matrix

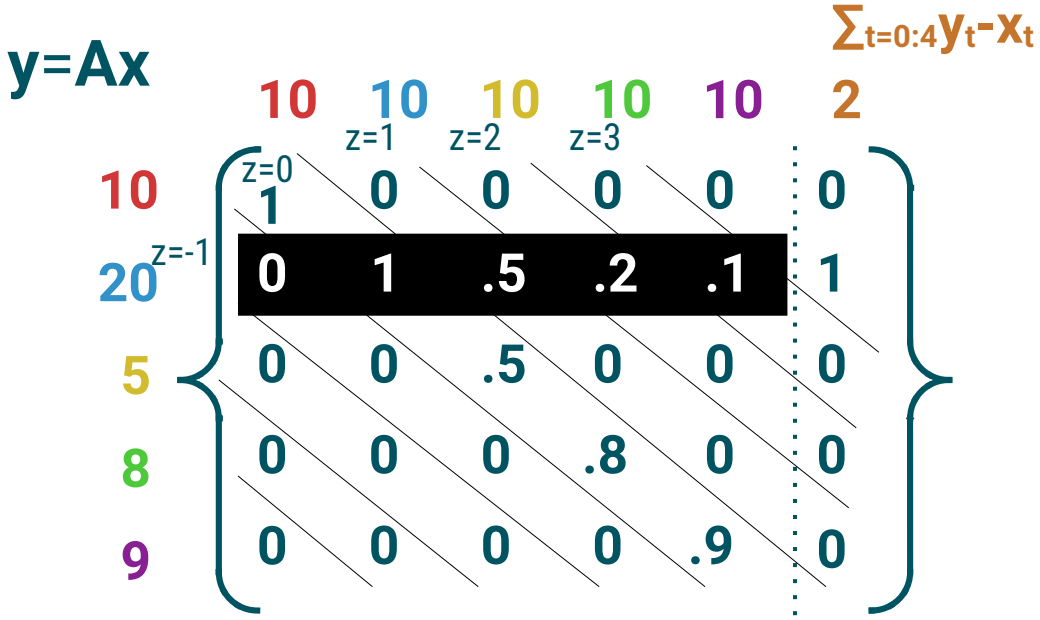
$y = Ax$

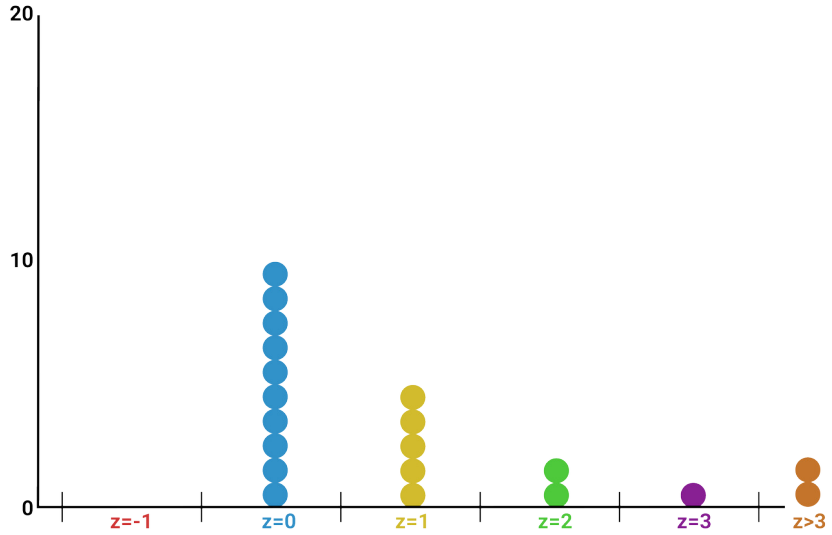
	10	10	10	10	10	$\sum_{t=0:4} y_t - x_t$
10	1	0	0	0	0	0
20	0	1	.5	.2	.1	1
5	0	0	.5	0	0	0
8	0	0	0	.8	0	0
9	0	0	0	0	.9	0



$$y_i = \sum_{j \text{ in } 1:6} x_j \, a_{ij}$$

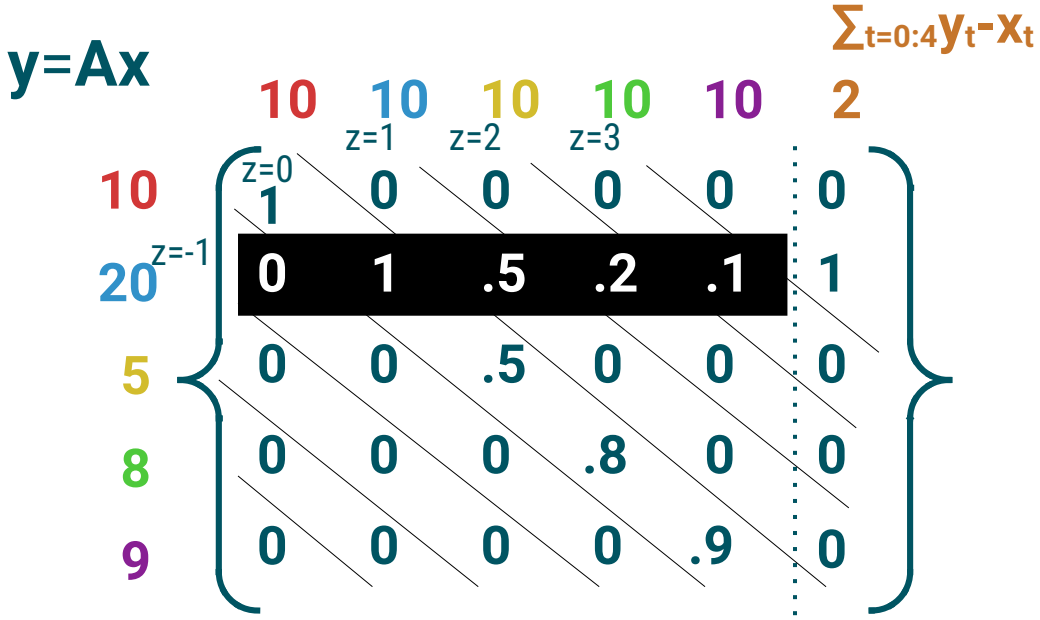
The displacement matrix

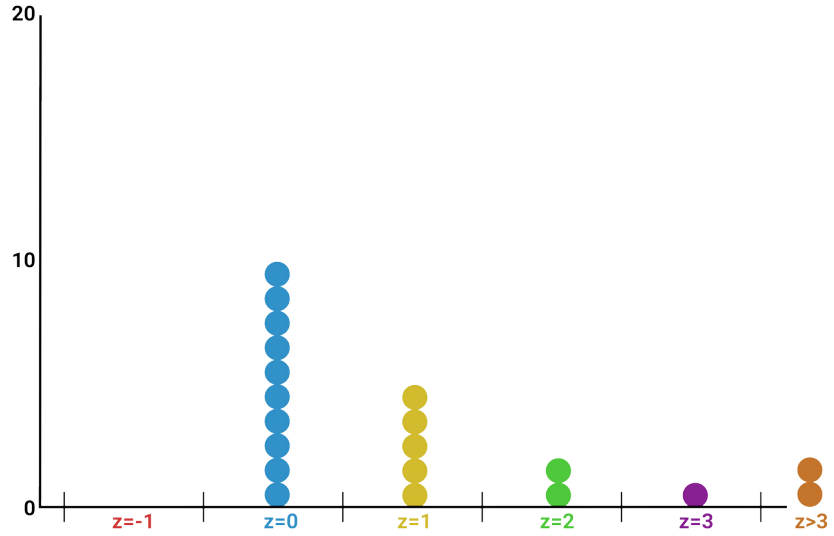




$$y_i = \sum_{j \text{ in } 1:6} x_j \, a_{ij}$$

The displacement matrix



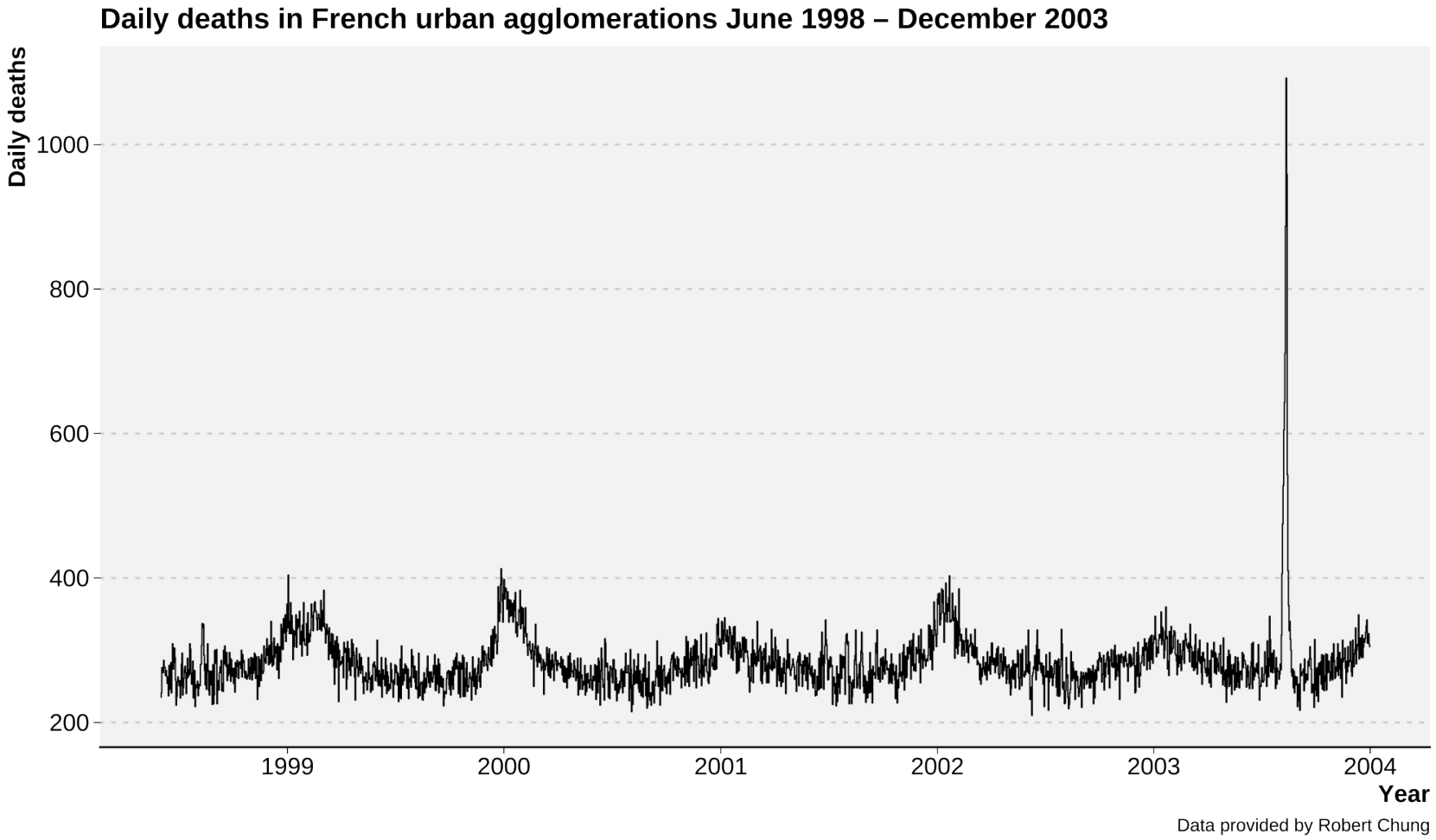


$$y_i = \sum_{j \text{ in } 1:6} x_j \, a_{ij}$$

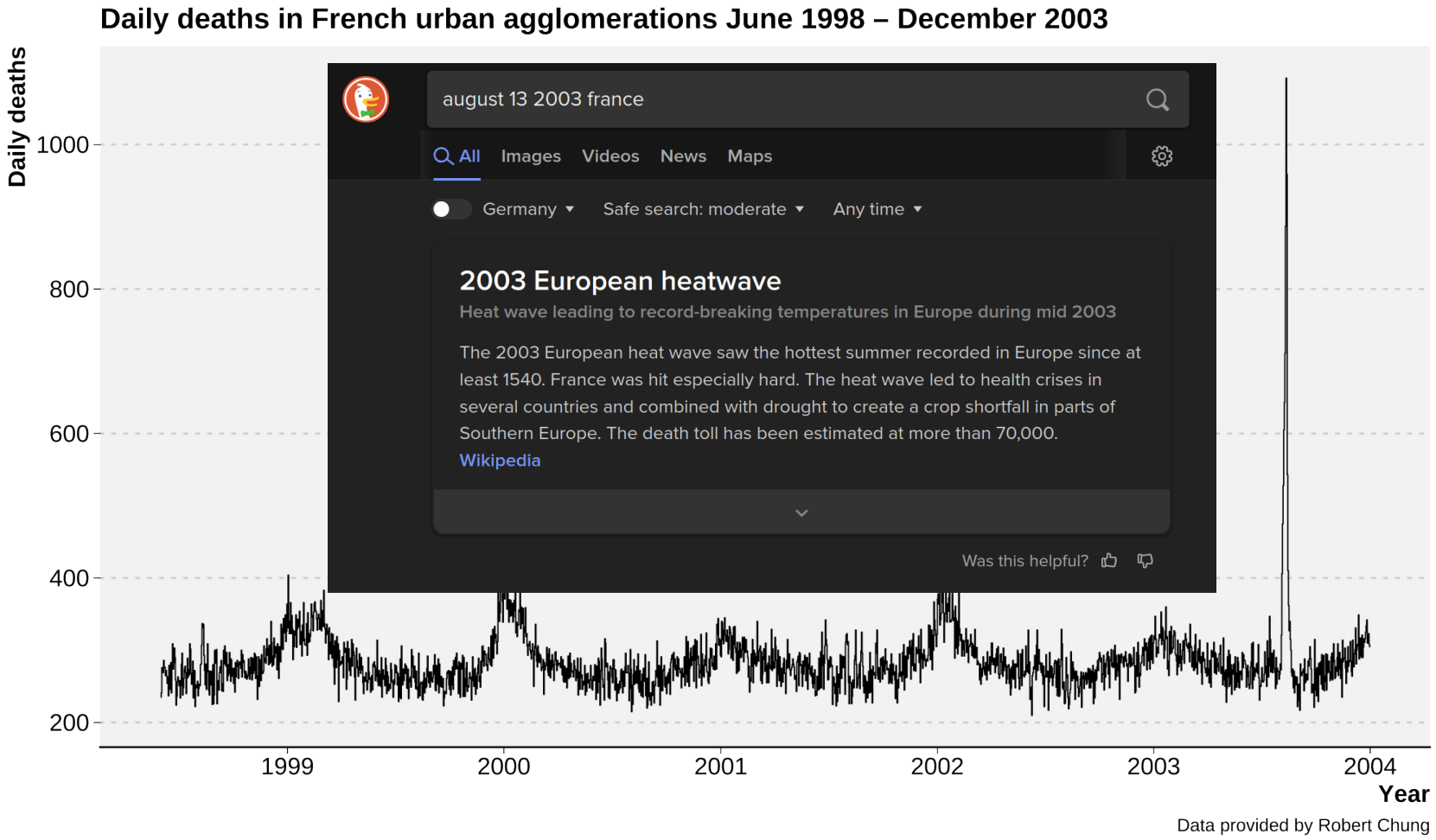
$$y_t = \sum_{z \text{ in } -1:4} x_{t+z} \, a_{t,t+z}$$

Mortality displacement during heatwaves

Mortality displacement during heatwaves

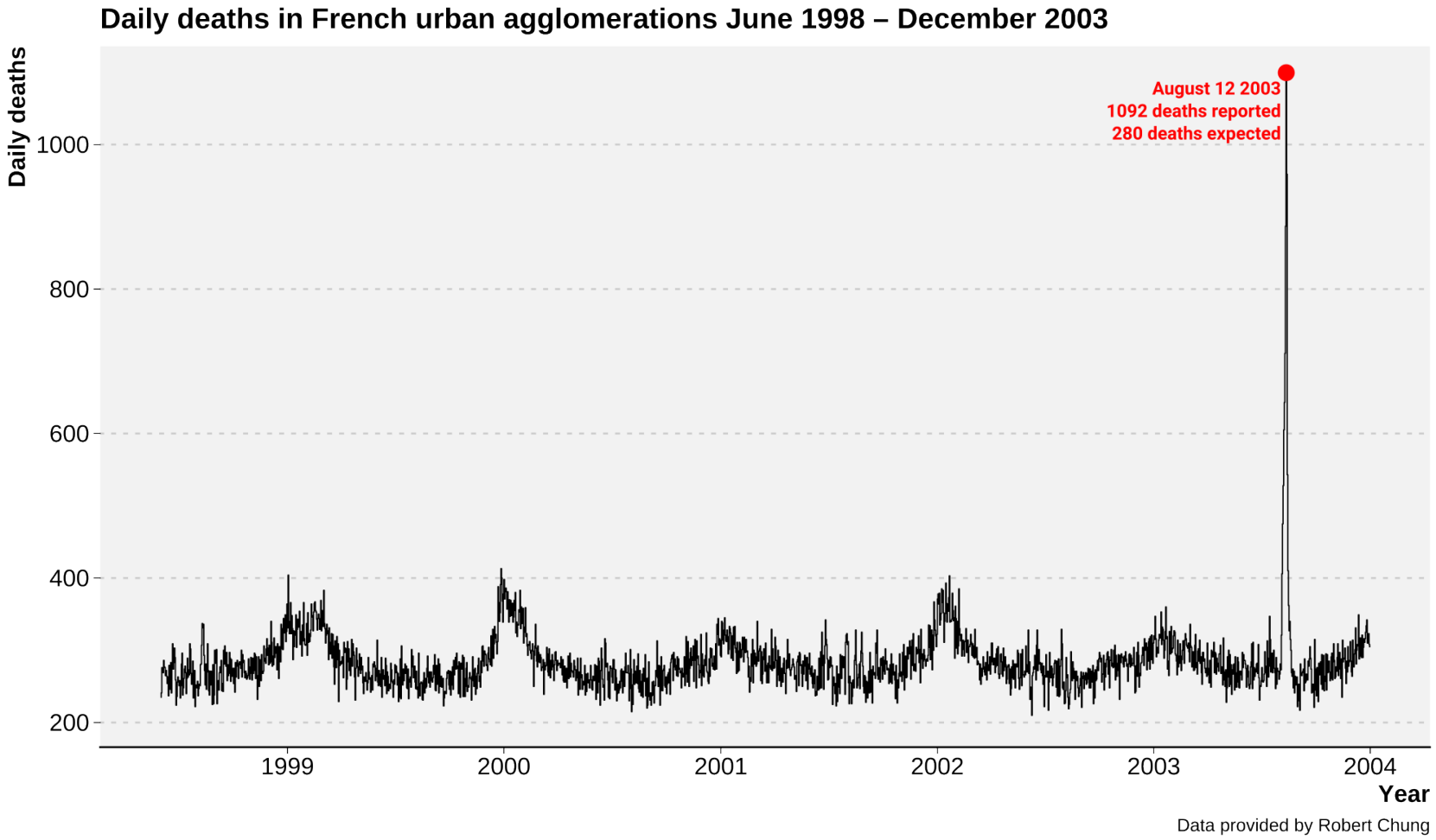


Mortality displacement during heatwaves

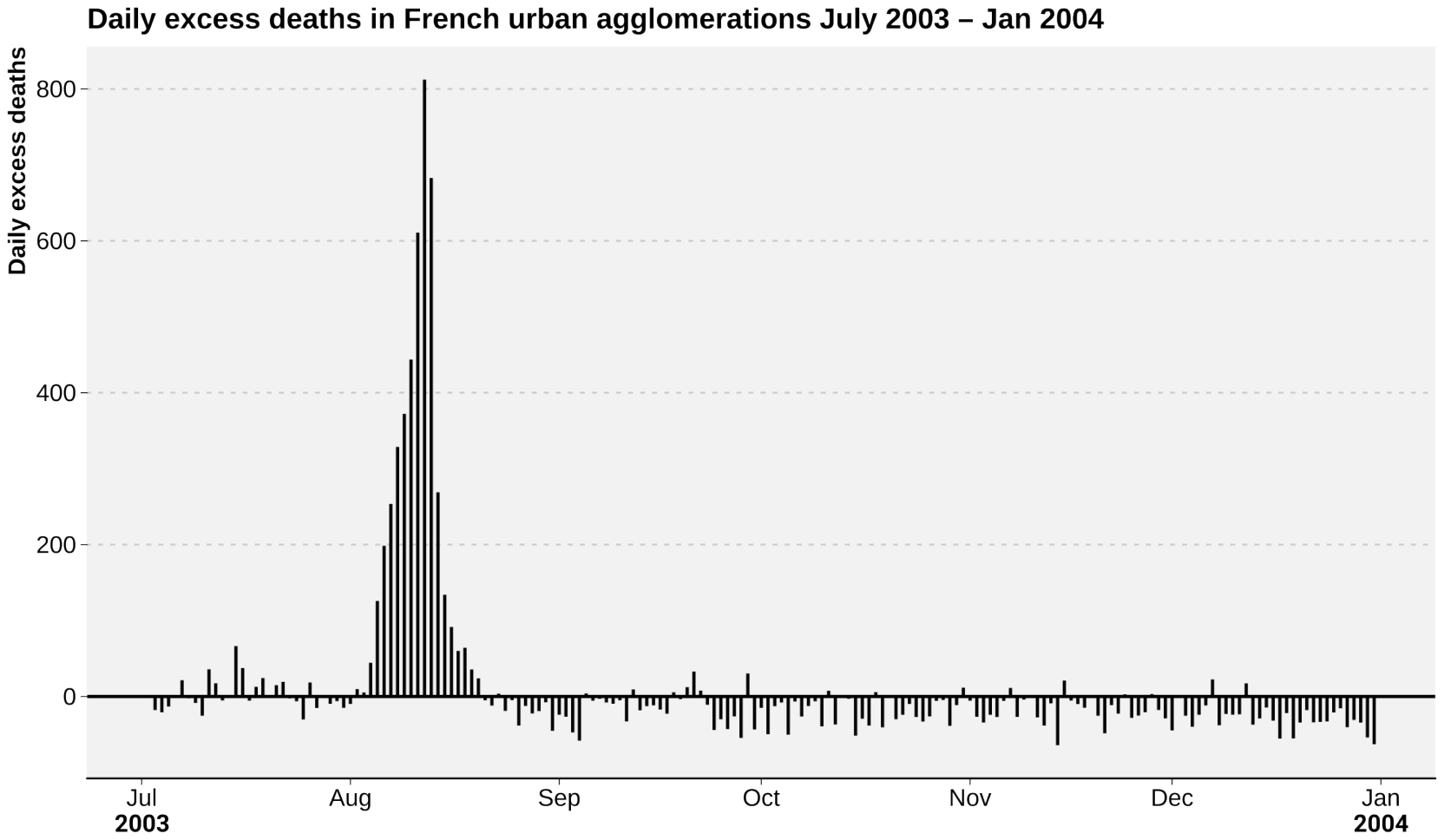


Data provided by Robert Chung

Mortality displacement during heatwaves



Mortality displacement during heatwaves



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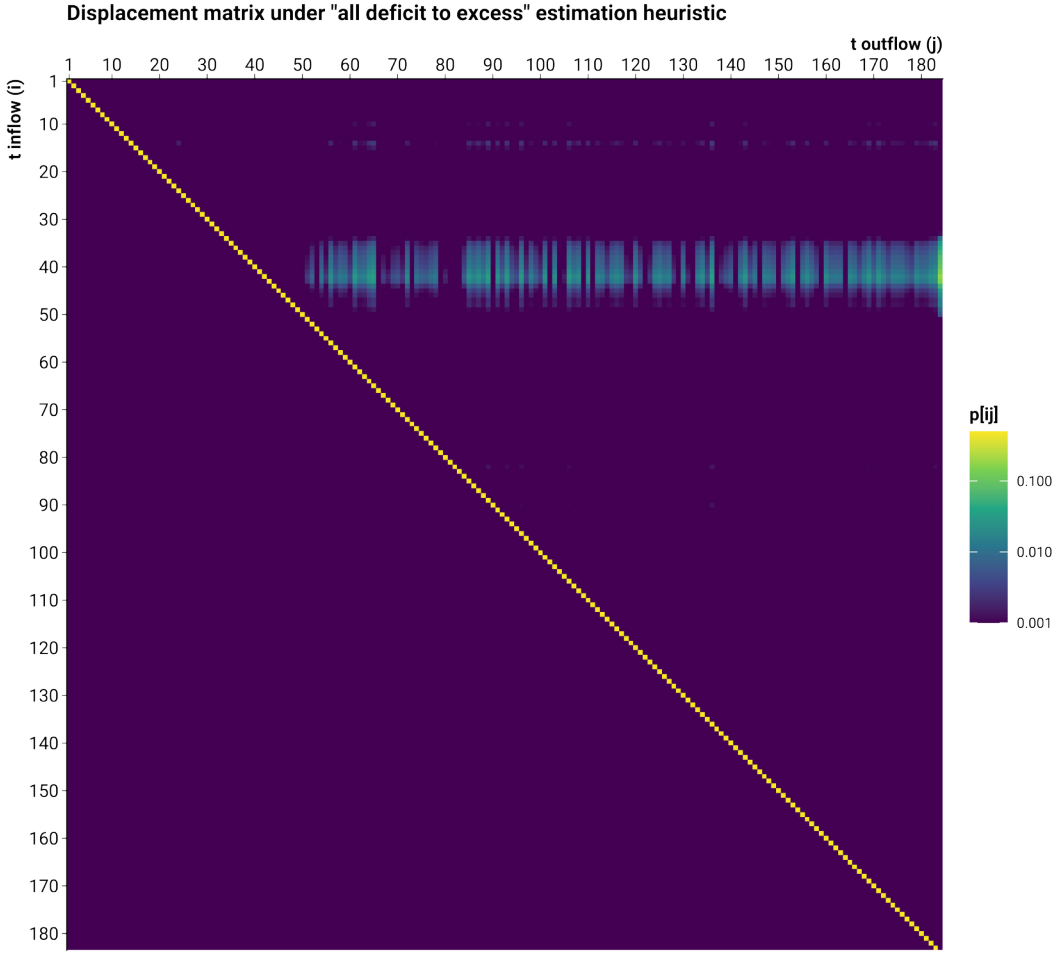
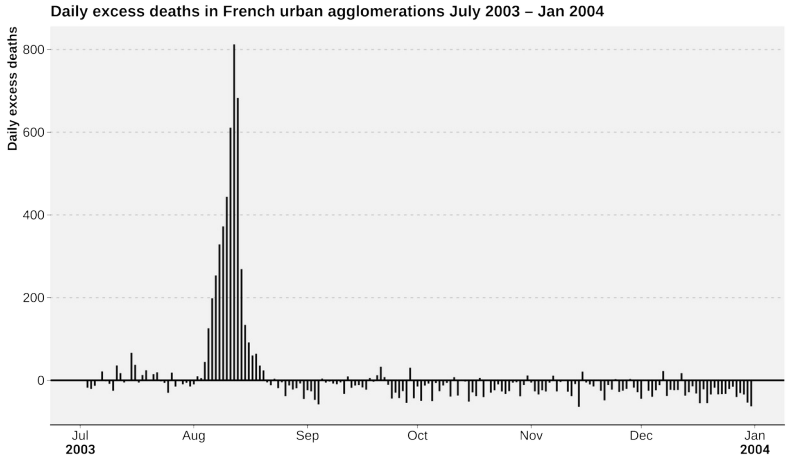
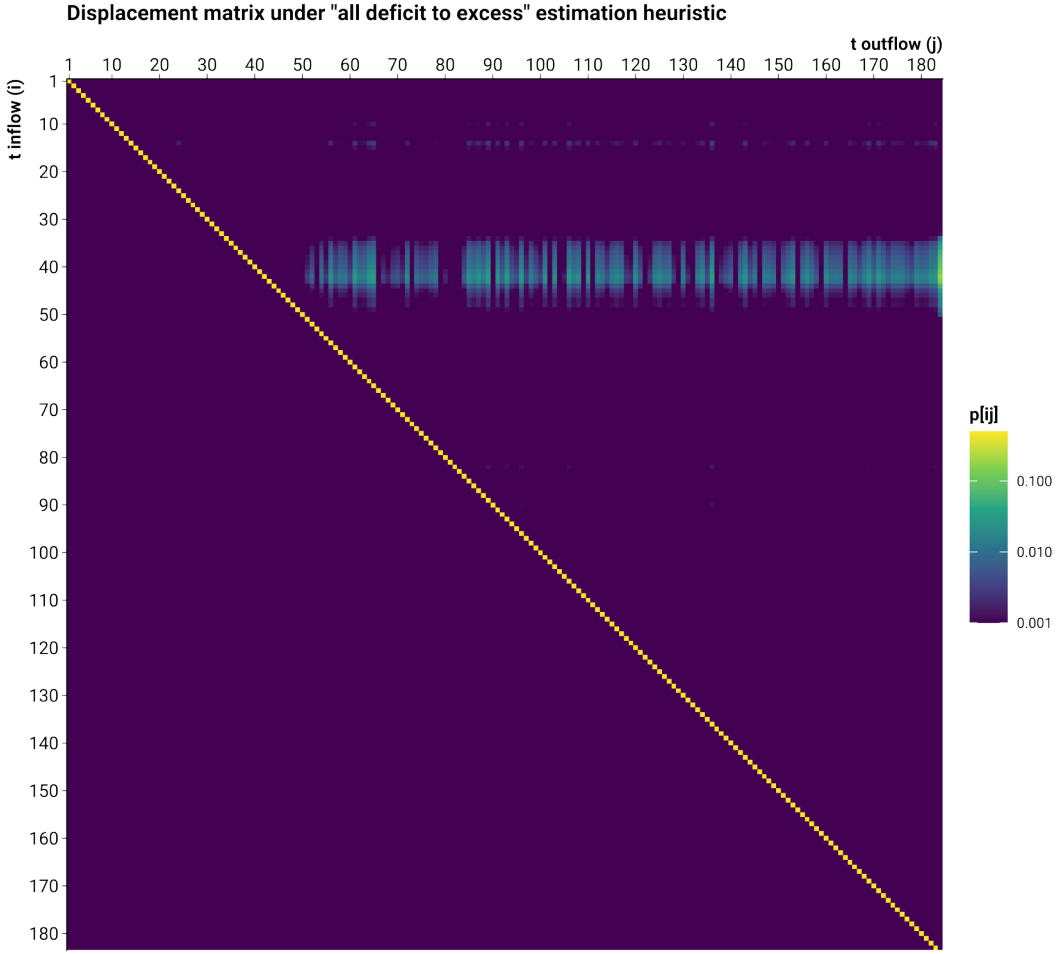
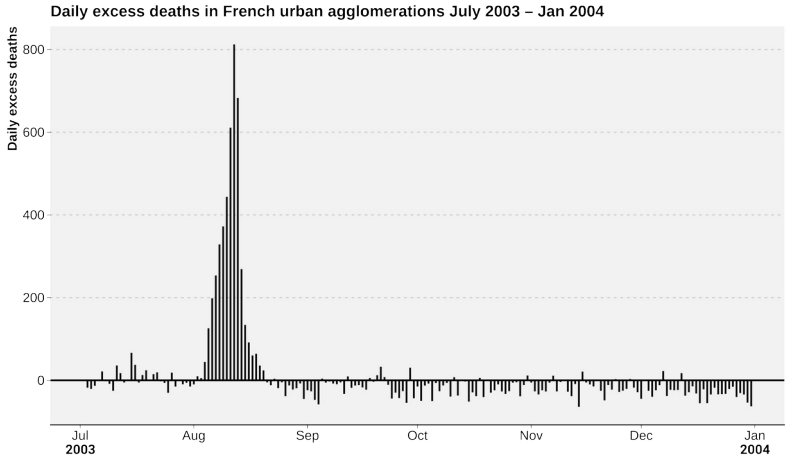
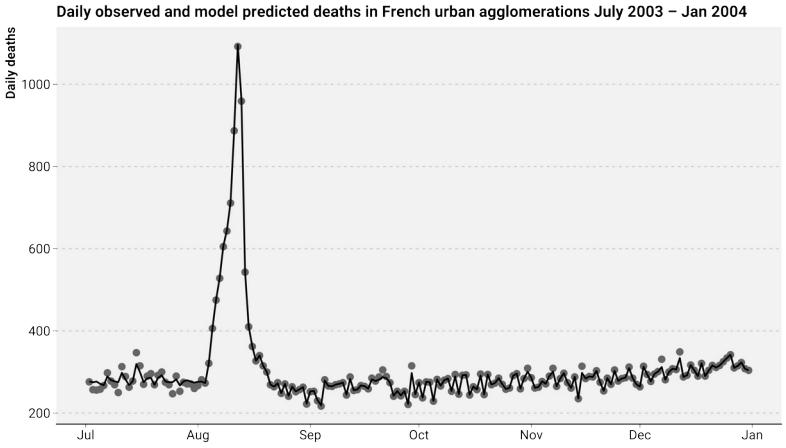




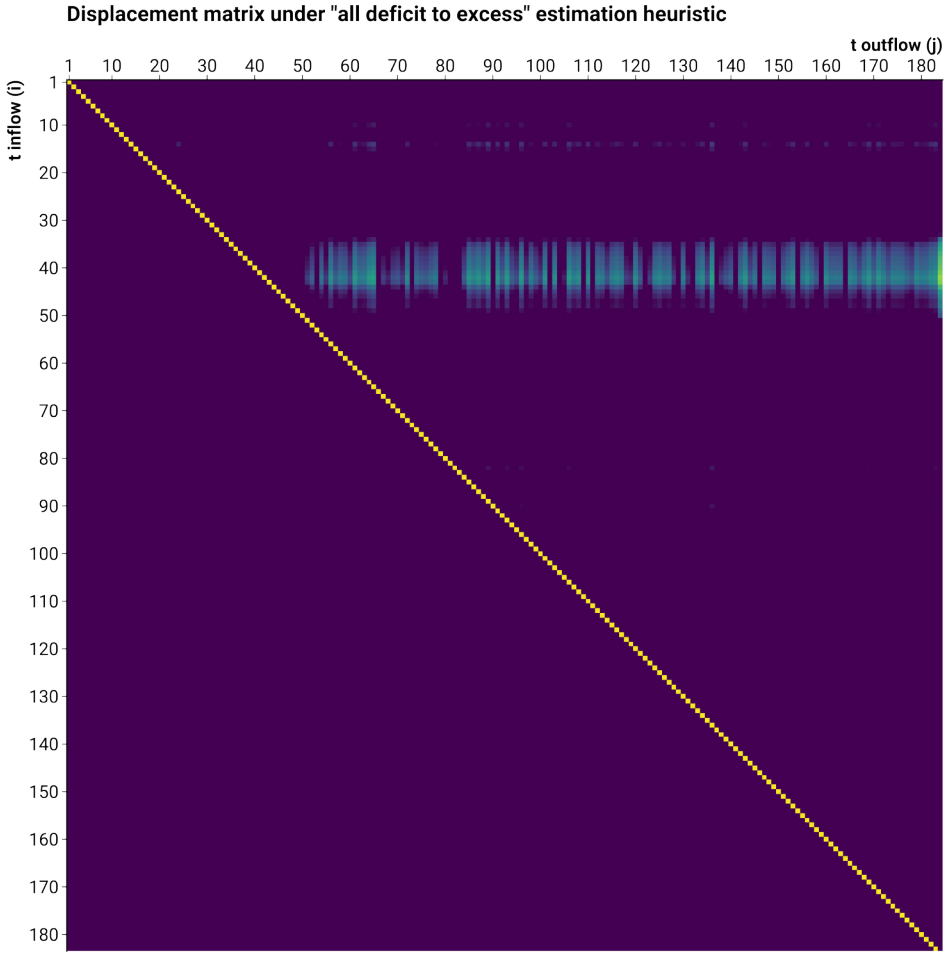
Mortality displacement during heatwaves

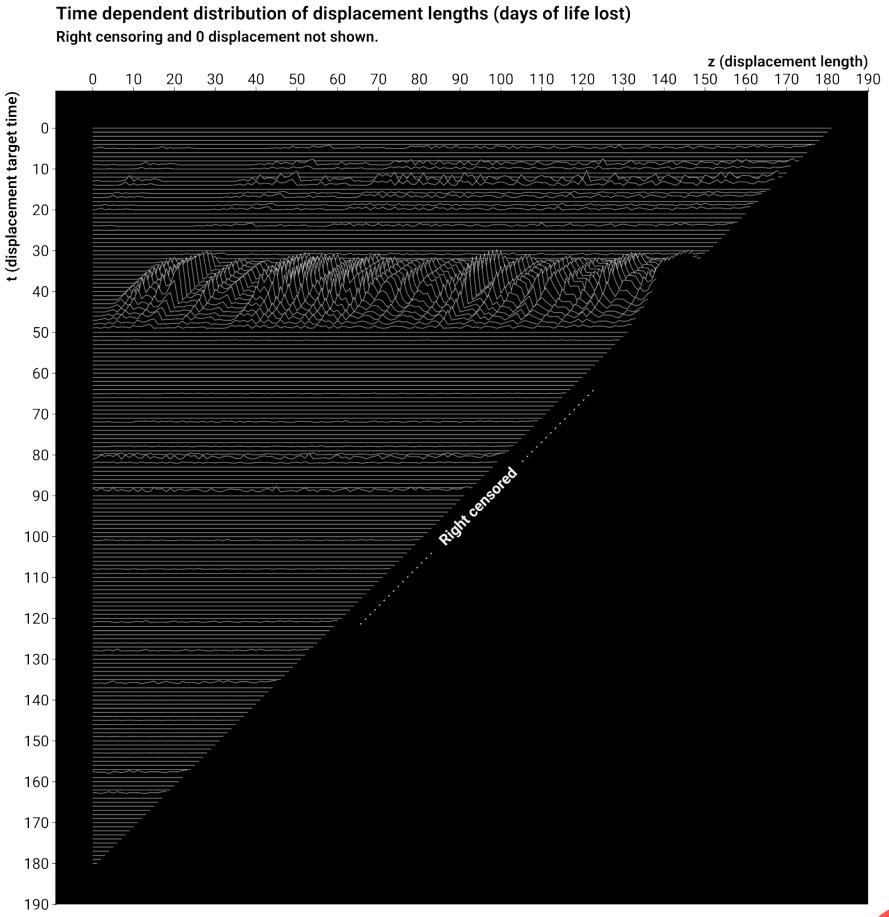




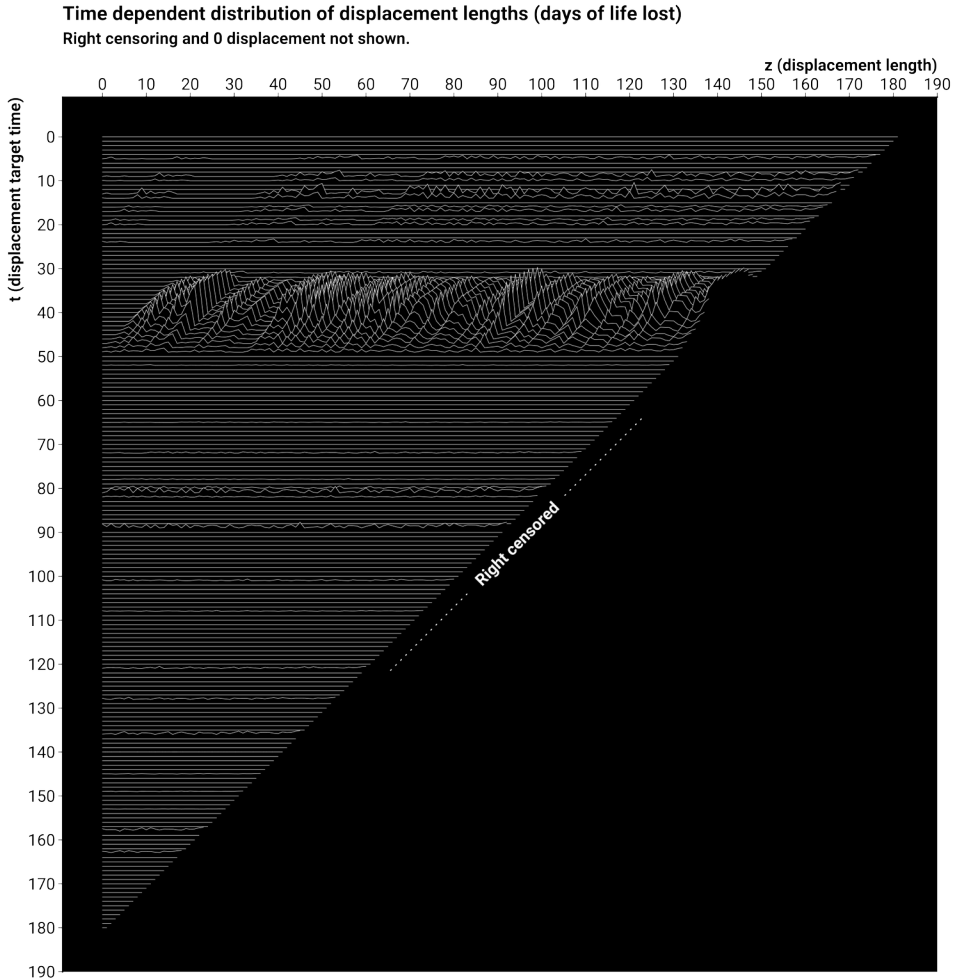


Mortality displacement during heatwaves

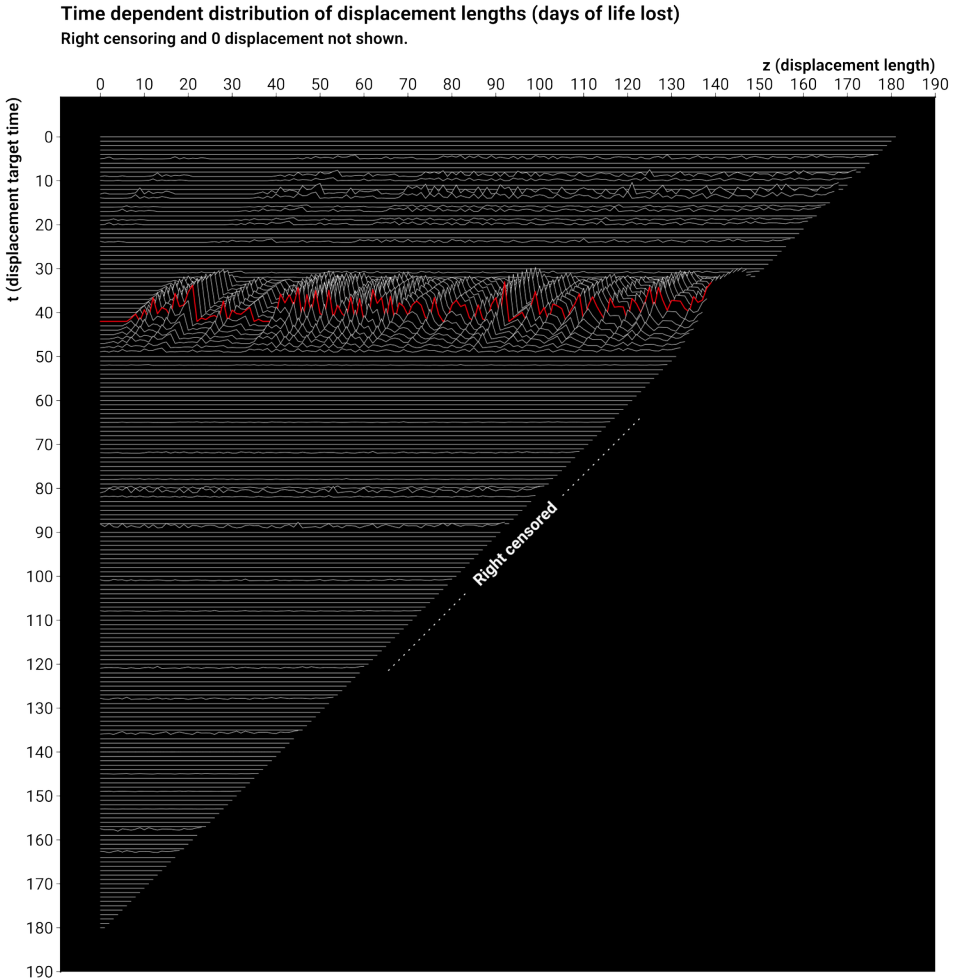




Mortality displacement during heatwaves



Mortality displacement during heatwaves



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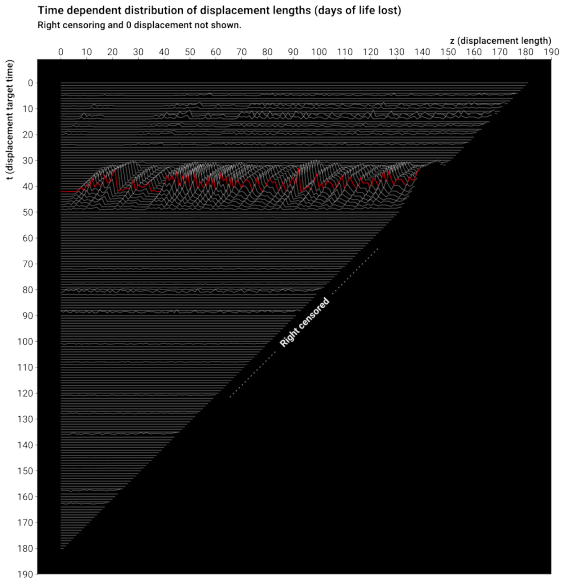

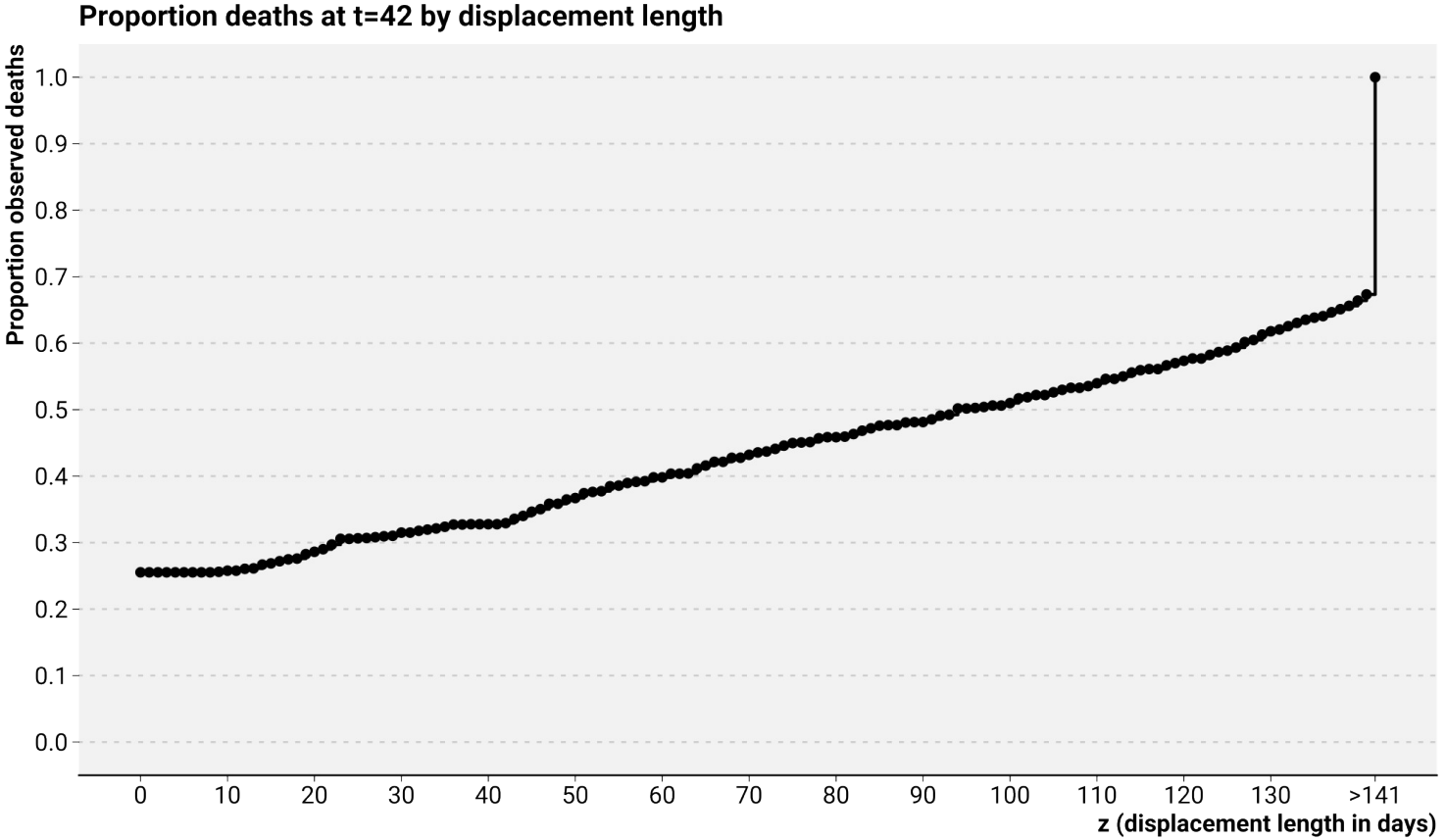

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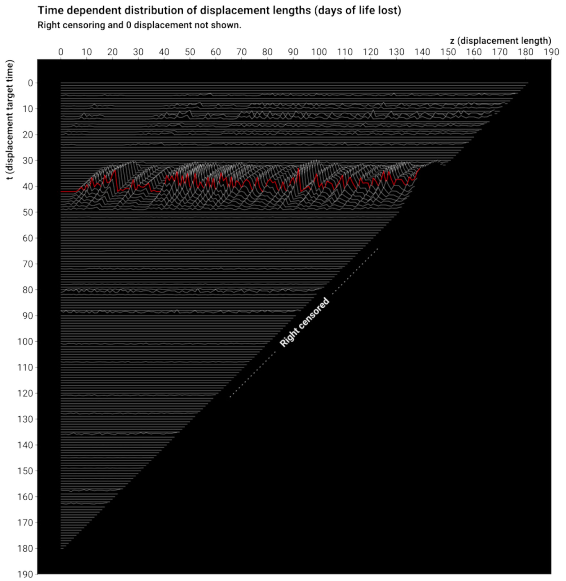

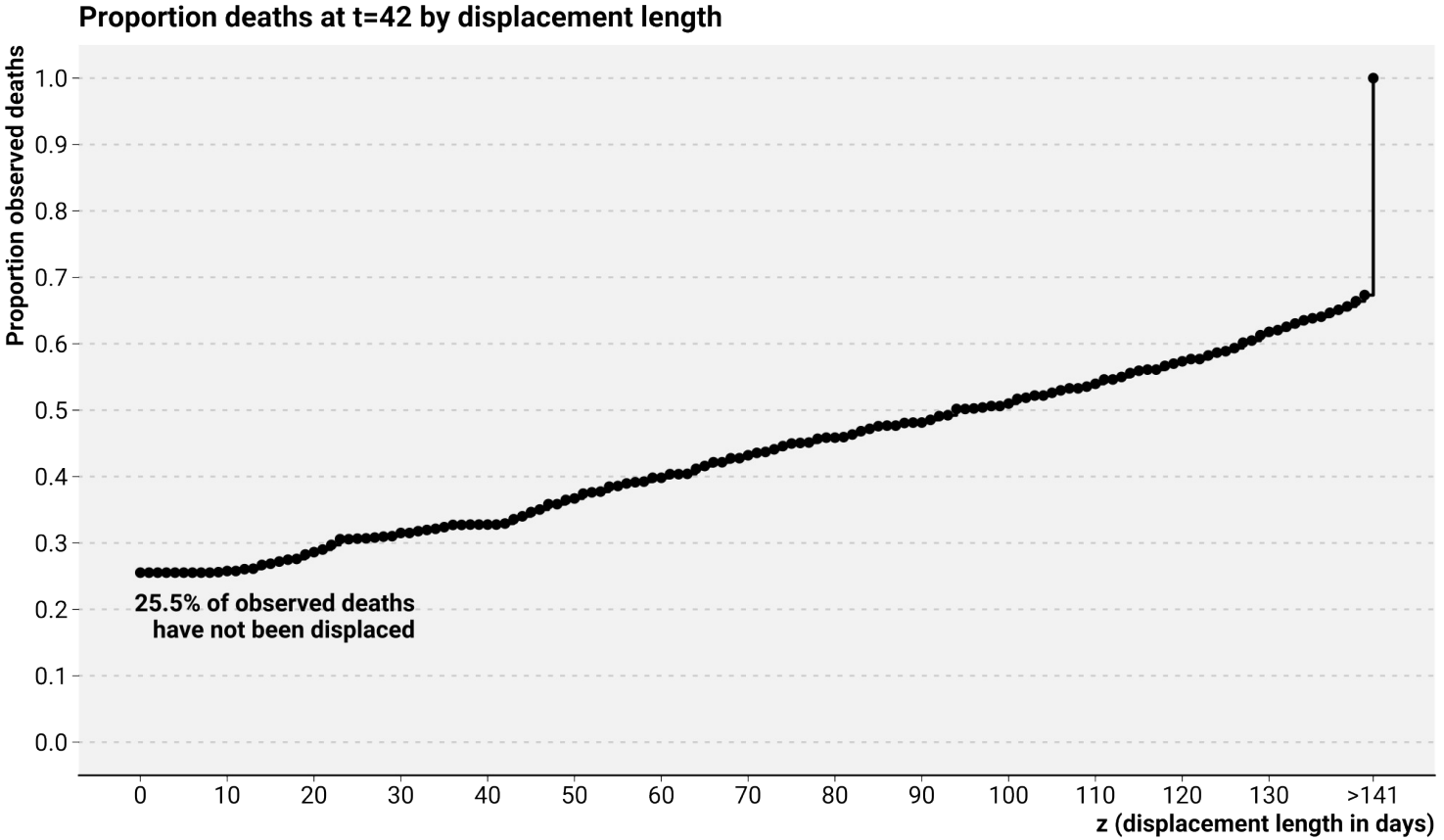

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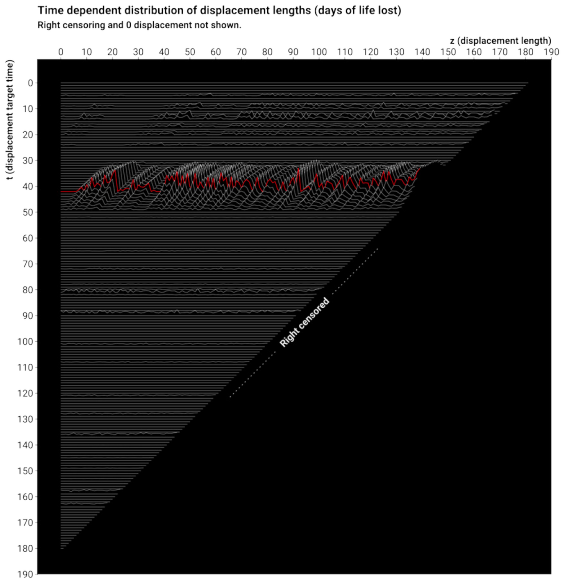

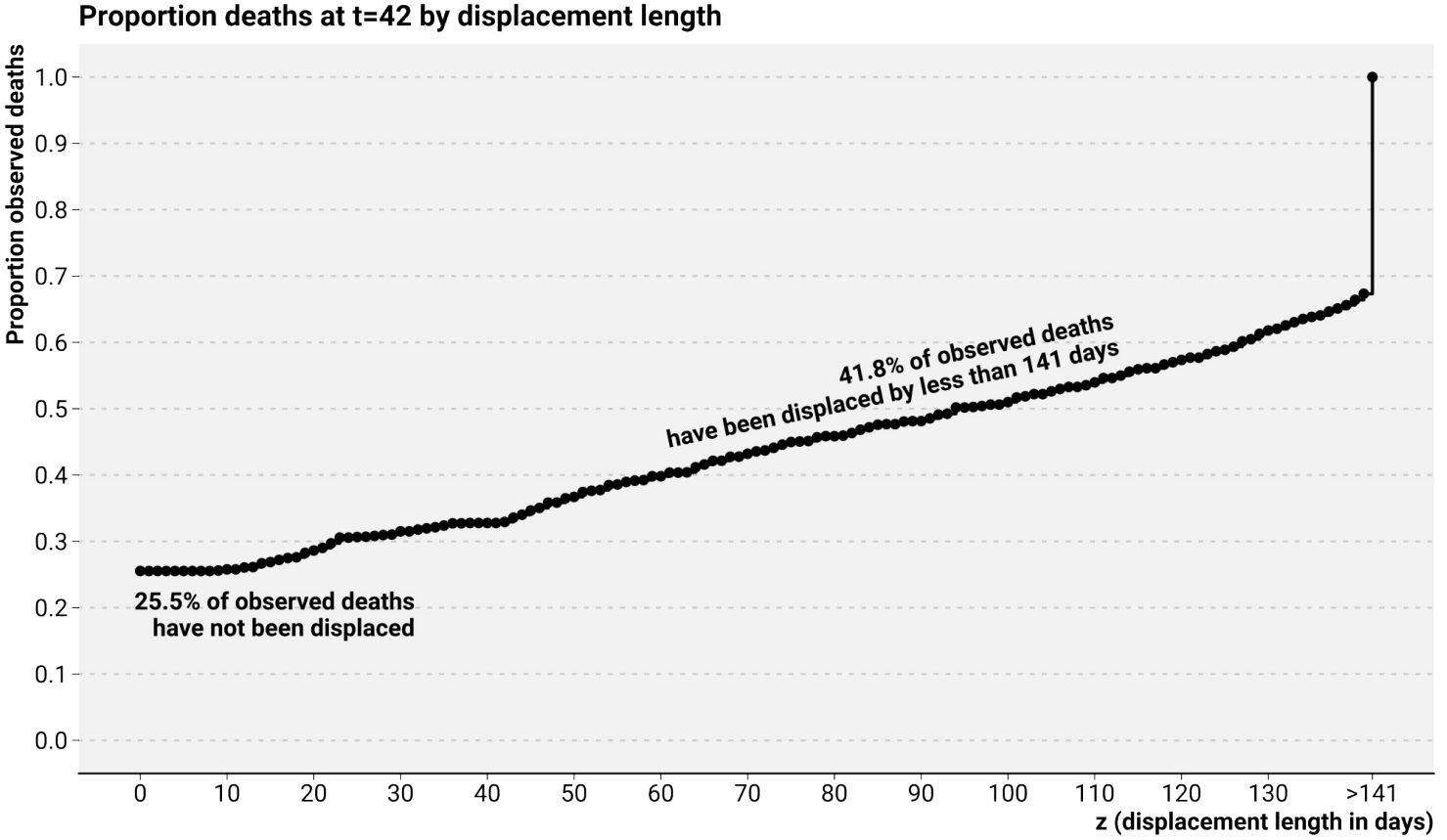

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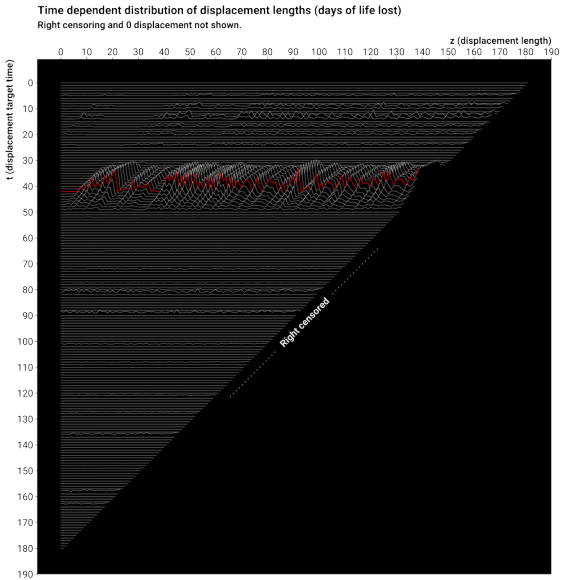

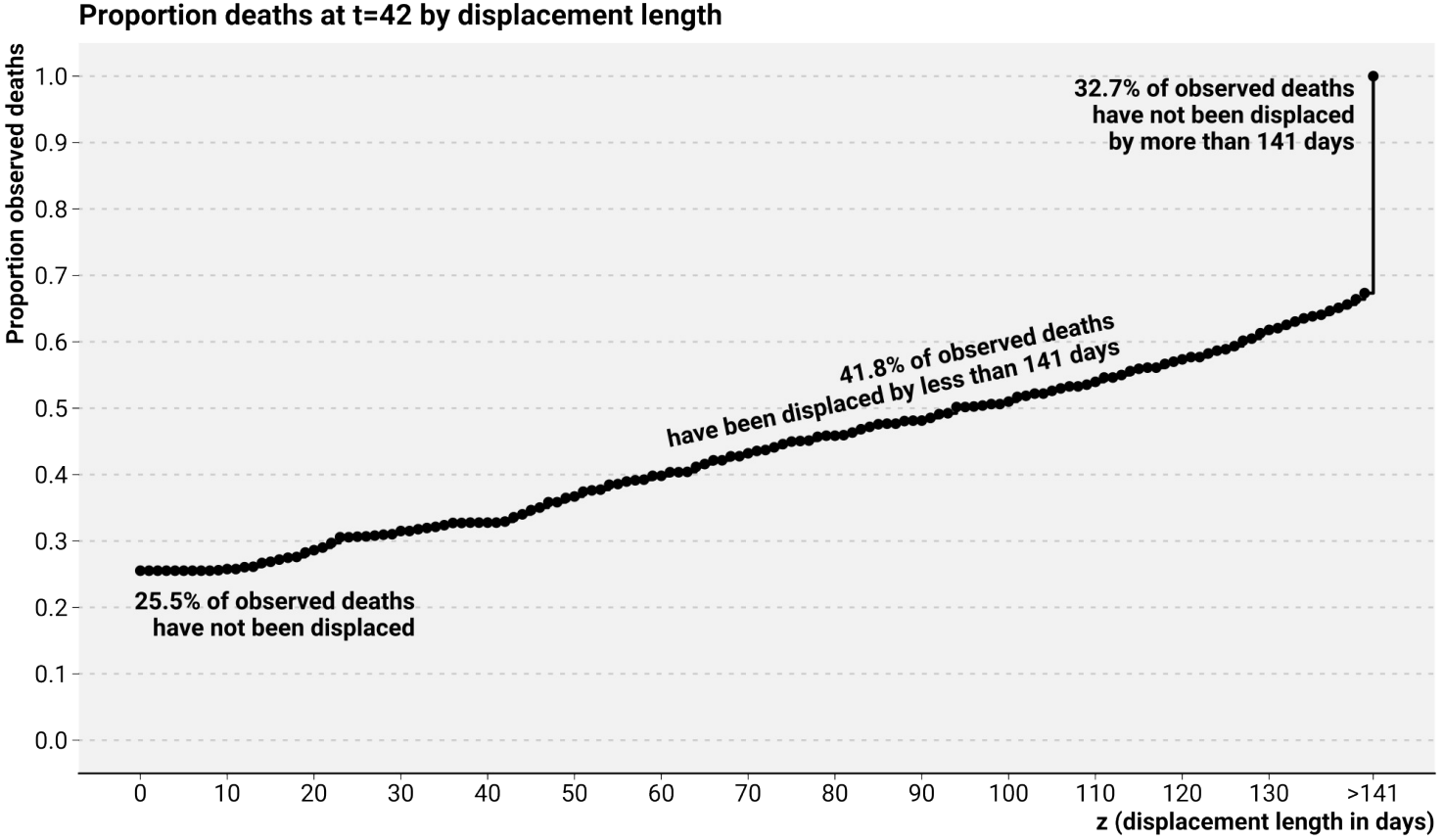

Convolution in survival analysis

Convolution in survival analysis

$$f_Y(y) = \int_0^{\omega} f_X(x = y+z) f_Z(z)\, dz$$

$$\mathbf{Y = X - Z}$$

Actual age at death = Age at death under expected scenario – Lifetime lost

Convolution in survival analysis

Independent displacement

$$f_Y(y) = \int_0^\omega f_X(x = y+z) f_Z(z)\, dz$$

$$\mathbf{Y = X - Z}$$

Actual age at death = Age at death under expected scenario – Lifetime lost

Age dependent displacement

$$f_Y(y) = \int_0^\omega \int_0^\omega f_X(x = y+z) f_{Z|X}(z|x)\, dz\, dx$$

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