

# The “birth hump”

## A shape decomposition of perinatal excess mortality

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MAX PLANCK INSTITUTE  
FOR DEMOGRAPHIC RESEARCH

# ontogenescence, noun

Valen, Leigh Van. 1975. "Life, Death, and Energy of a Tree."

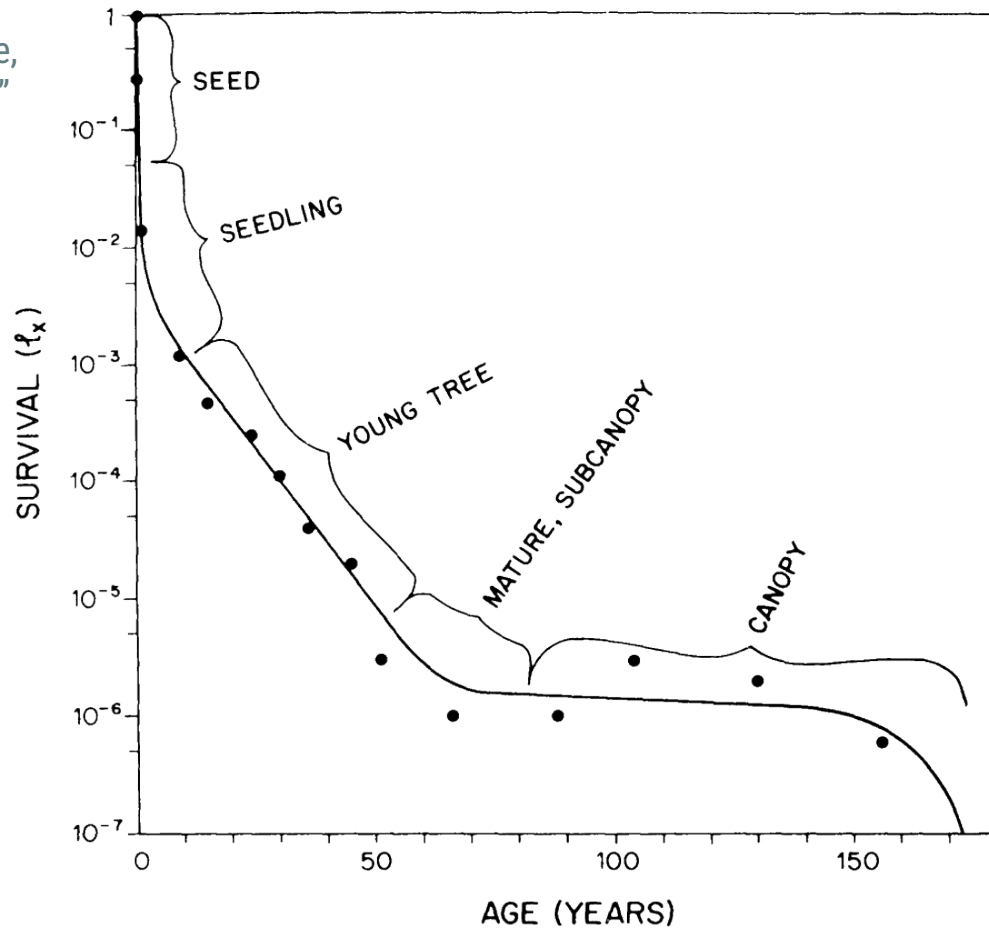
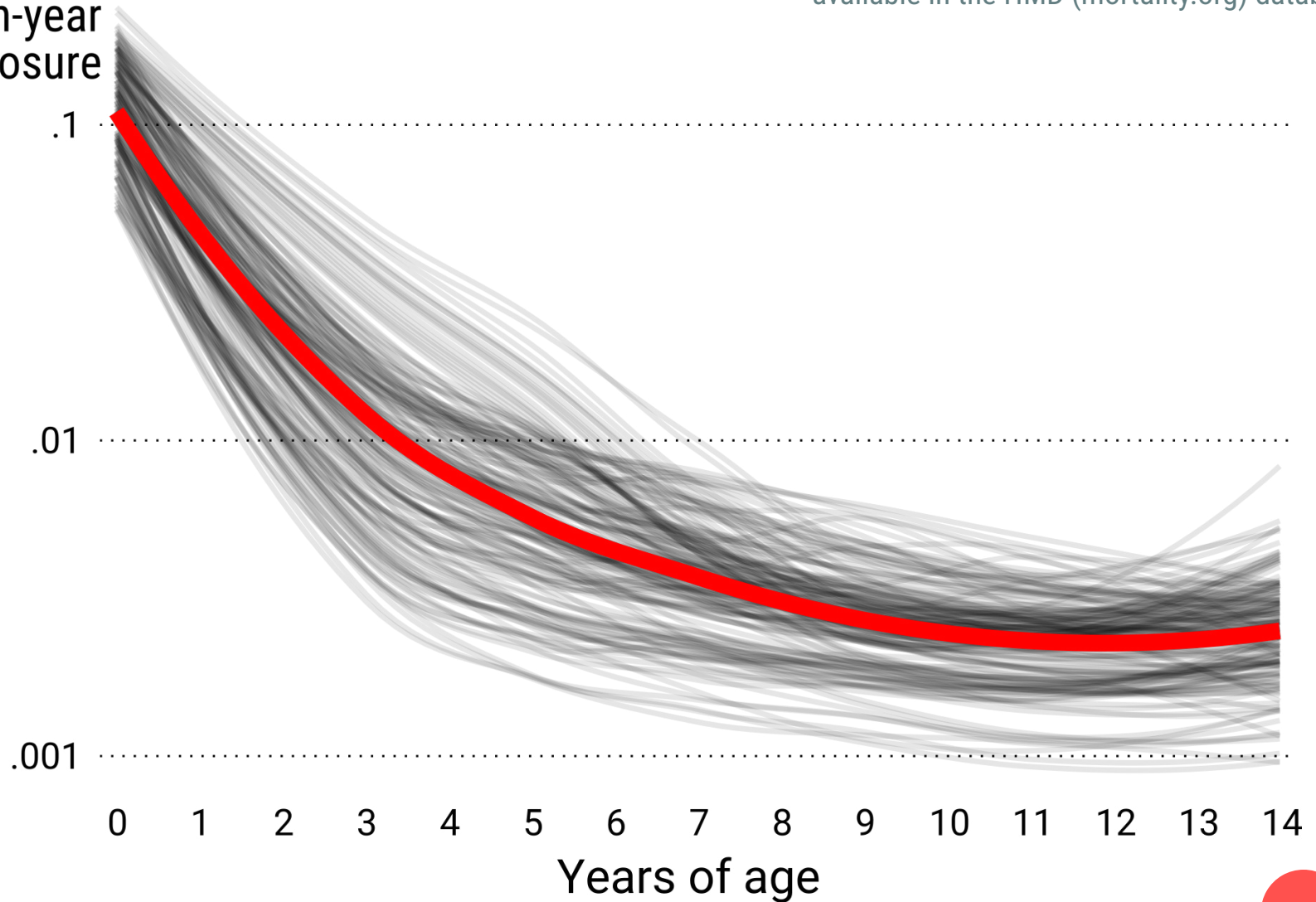


FIGURE 1. Survivorship curve for *Euterpe globosa*, over 7 orders of magnitude. All values of  $l_x$  except that at 9 years are completely independent of each other, thus providing an internal check on accuracy.

# onto|gen|ecence, noun

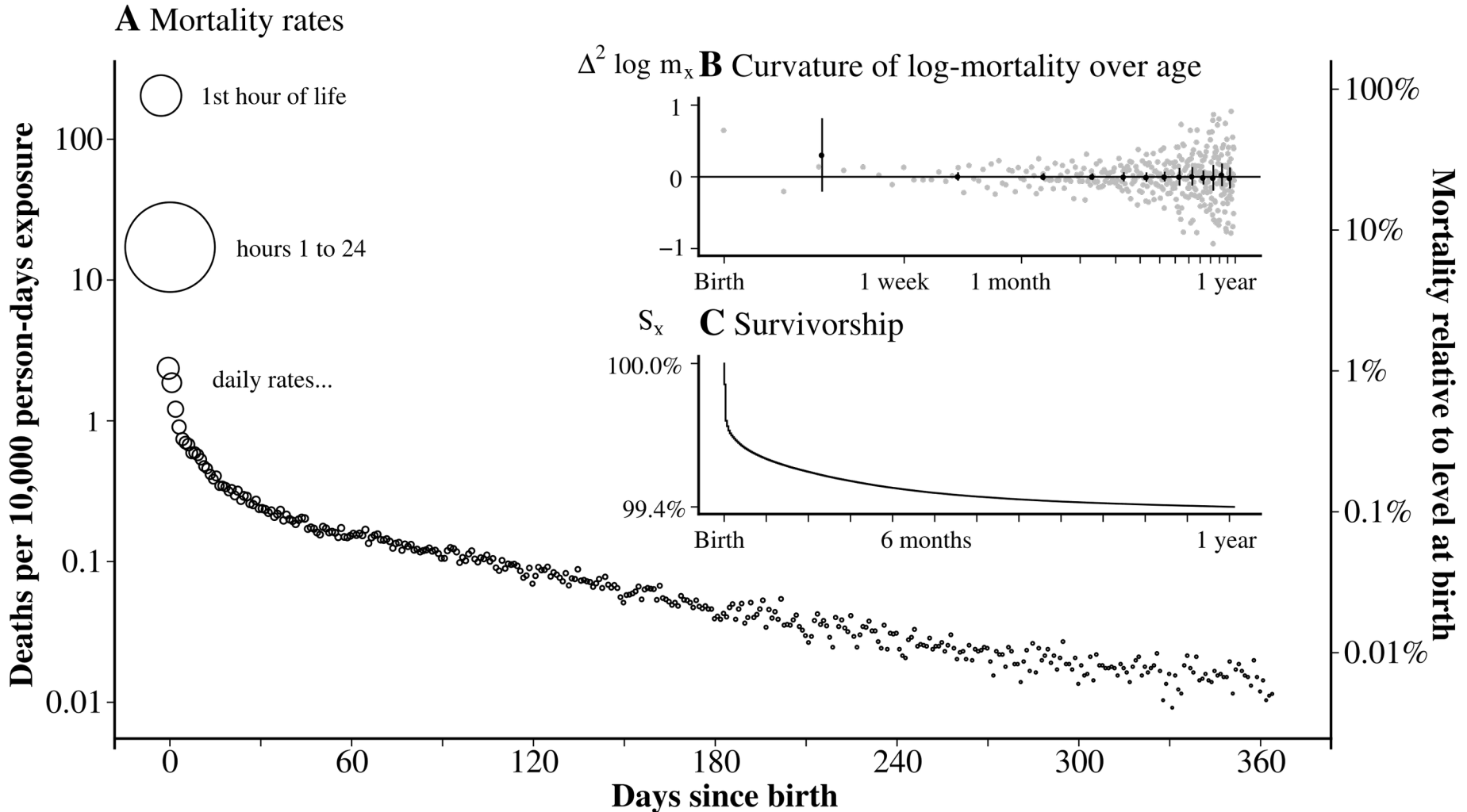
Deaths per  
person-year  
of exposure

LOESS-smoothed mortality rates for various populations  
available in the HMD (mortality.org) database.



**The formal demography  
of ontogenescence  
In search of parsimony**

US birth cohort 2005–2010



# ontogenescence, noun

*Review. Mortality before senescence* D. A. Levitis 805

Table 1. Primary hypotheses for the evolutionary basis of ontogenescence.

hypothesis name	hypothesis: death rate decreases with age in developing cohorts because. . .
quality control hypothesis	. . . kin terminate potentially inviable individuals early so as to avoid bearing unnecessary costs.
growth trade-off hypothesis	. . . as individuals grow, they can decrease their relative need for continued growth and therefore accept fewer growth-enabling mortality risks.
robustness hypothesis	. . . as individuals develop, they acquire characteristics that increase their robustness to insults.
heterogeneous frailty hypothesis	. . . the frailest individuals die first, causing mean frailty to decline with age.
transitional timing hypothesis	. . . transcriptional, developmental and environmental transitions are dangerous, and these are concentrated early in life.

Levitis (2011). "Before senescence: the evolutionary demography of ontogenesis".

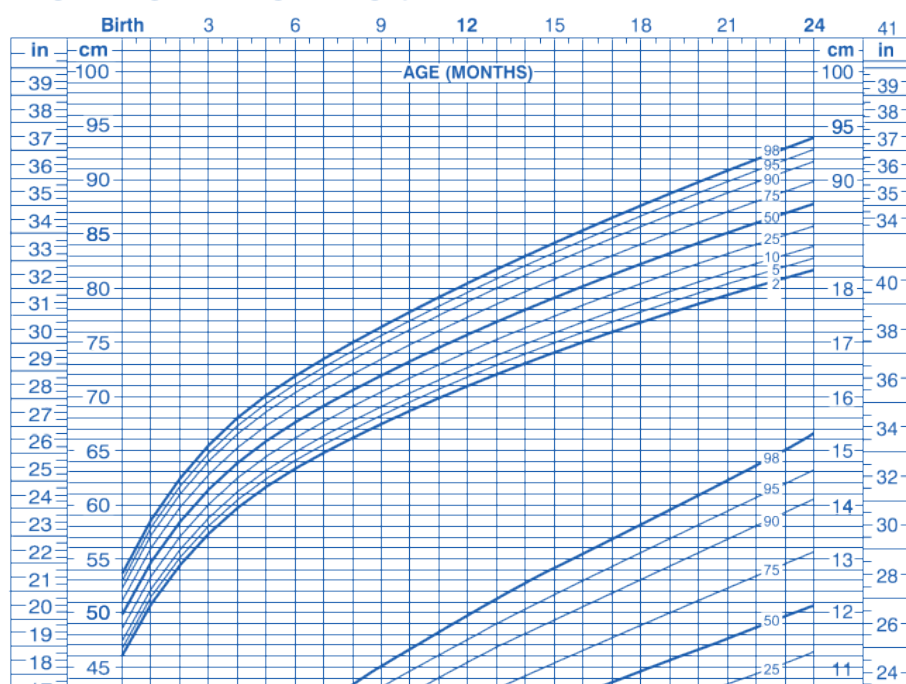
# ontogenecence as mortality adaptation

robustness hypothesis

... as individuals develop, they acquire characteristics that increase their robustness to insults.

Birth to 24 months: Boys

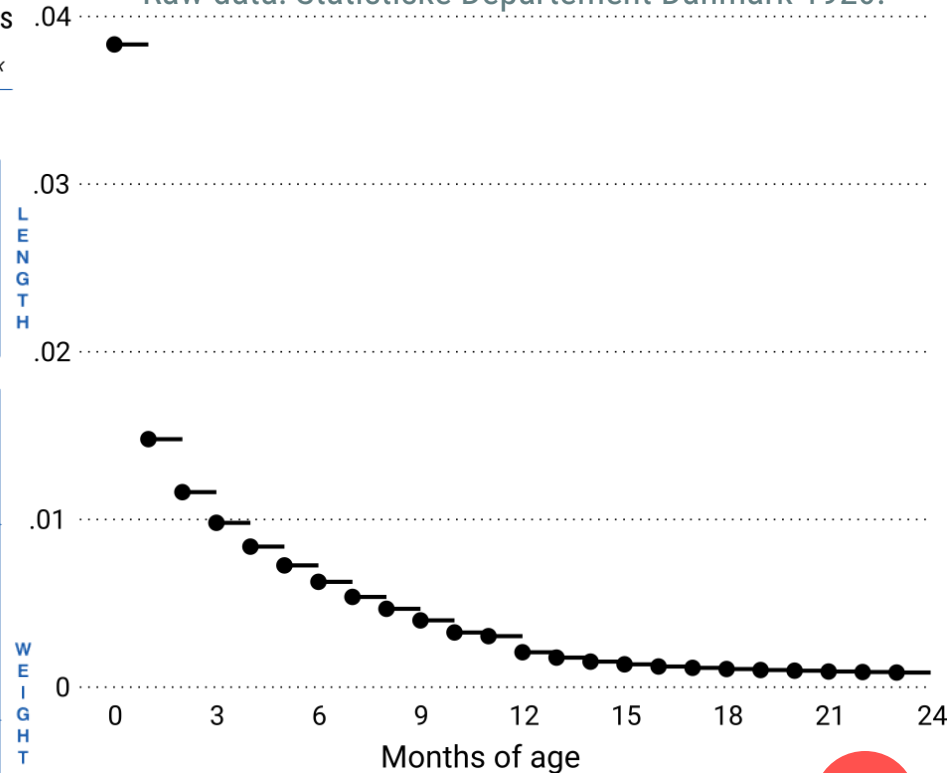
Length-for-age and Weight-for-age percentiles



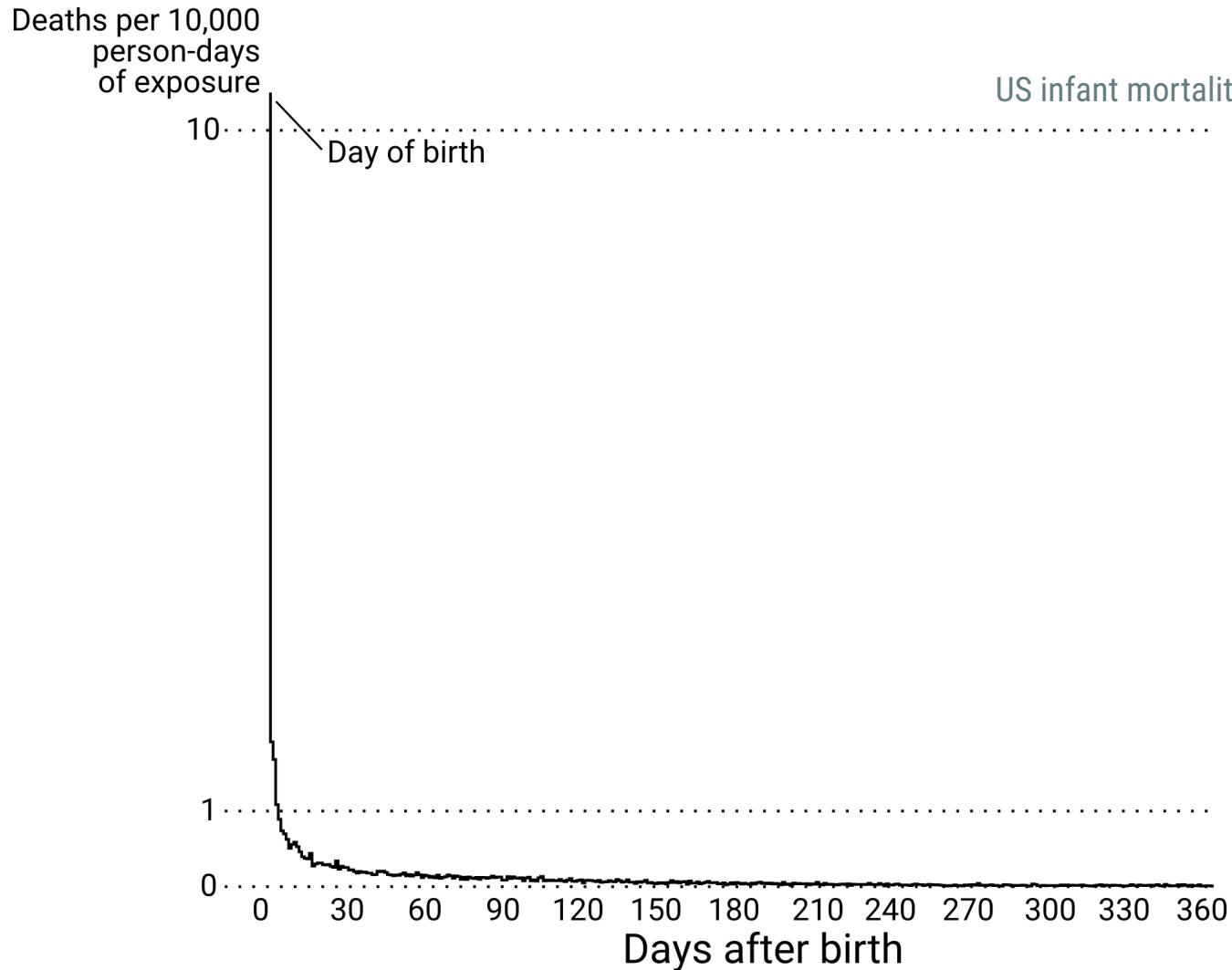
Probability to die within 30 days  ${}_1q_x$

Danish males born 1911–15.

Raw data: Statistiske Departement Danmark 1920.



# ontogen|ecence as mortality adaptation

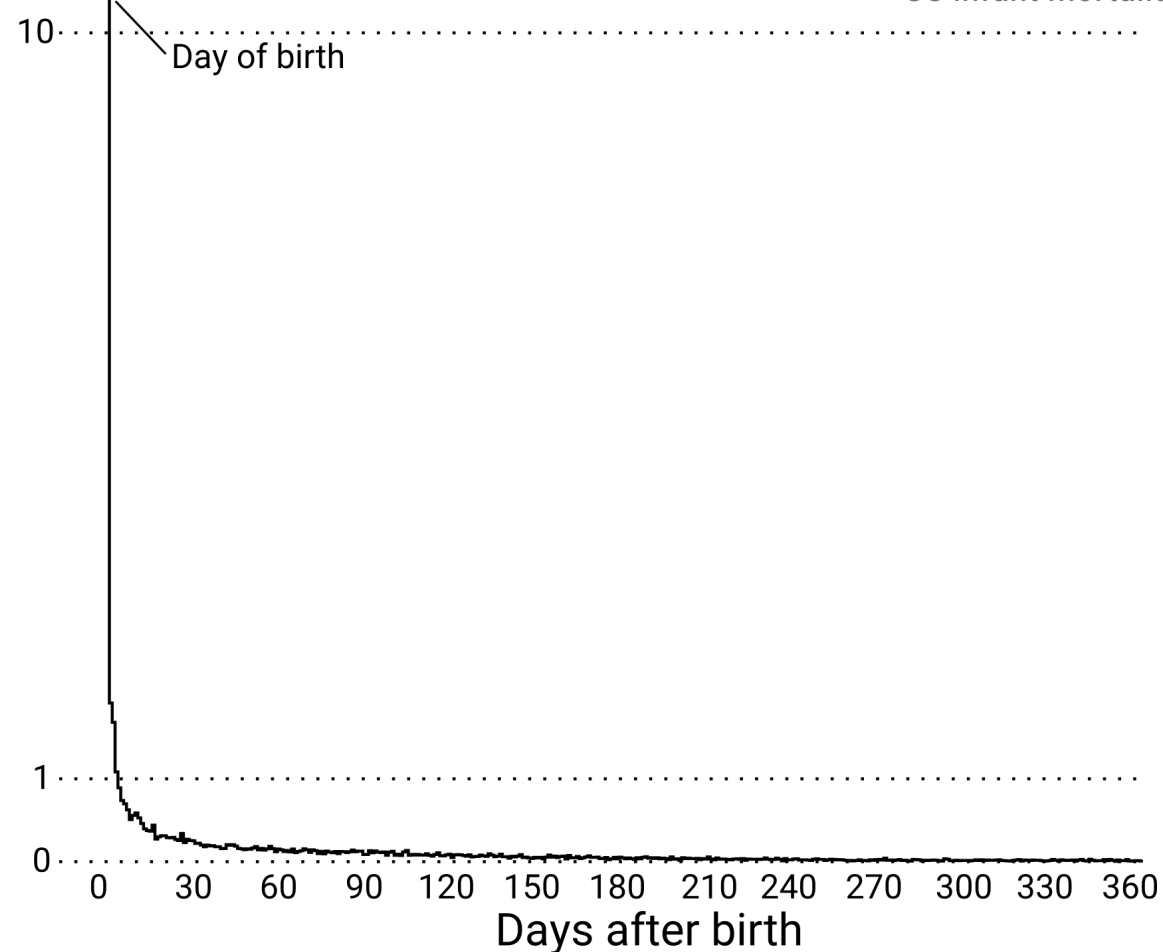


# ontogenetic mortality selection

heterogeneous frailty hypothesis

... the frailest individuals die first, causing mean frailty to decline with age.

Deaths per 10,000 person-days of exposure



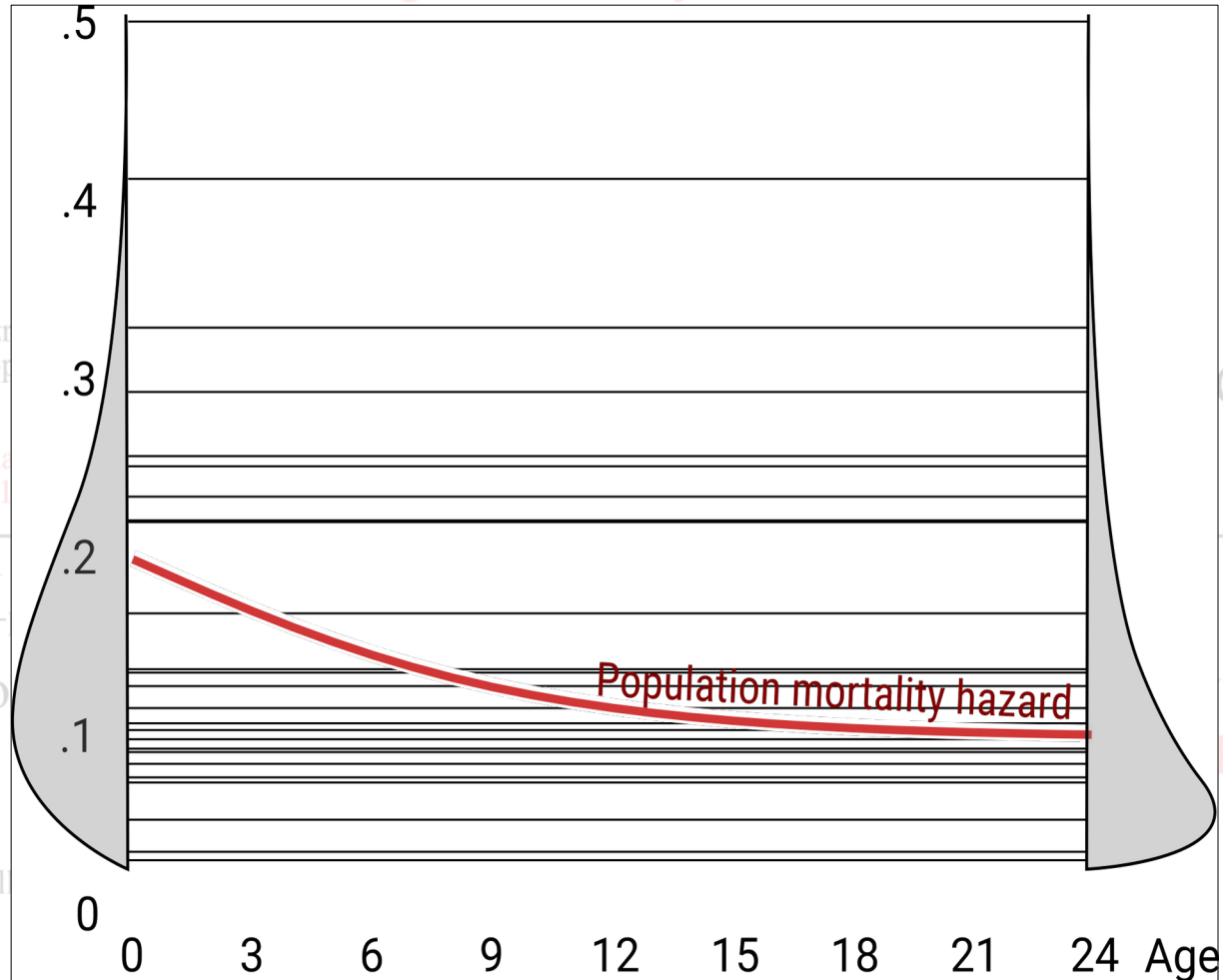
US infant mortality by day of age for children conceived in 2009.

Raw data: CDC/NCHS.

# ontogenetic mortality selection

heterogeneous frailty hypothesis

... the frailest individuals die first, causing mean frailty to decline



$bk(x+c)^{k-1}$   
Shifted-Weibull hzrd  
 $k=1-p$  and  $b=a/(1-p)$

$\delta(x+\theta)^{\alpha-1}$   
Hougaard-Exponential  
 $\delta=a, \theta=c$  and  $\alpha=1$

$c=1$

$a(x+1)^{-p}$

Unity-shifted-poisson



$bk(x+1)^{k-1}$   
Unity-shifted-Weibull

$ae^{-bx}$   
Gompertz hzrd

$p=0.5$

$a(x+c)^{-0.5}$   
Inverse-Gaussian-poisson hzrd.

# ontology of mortality selection

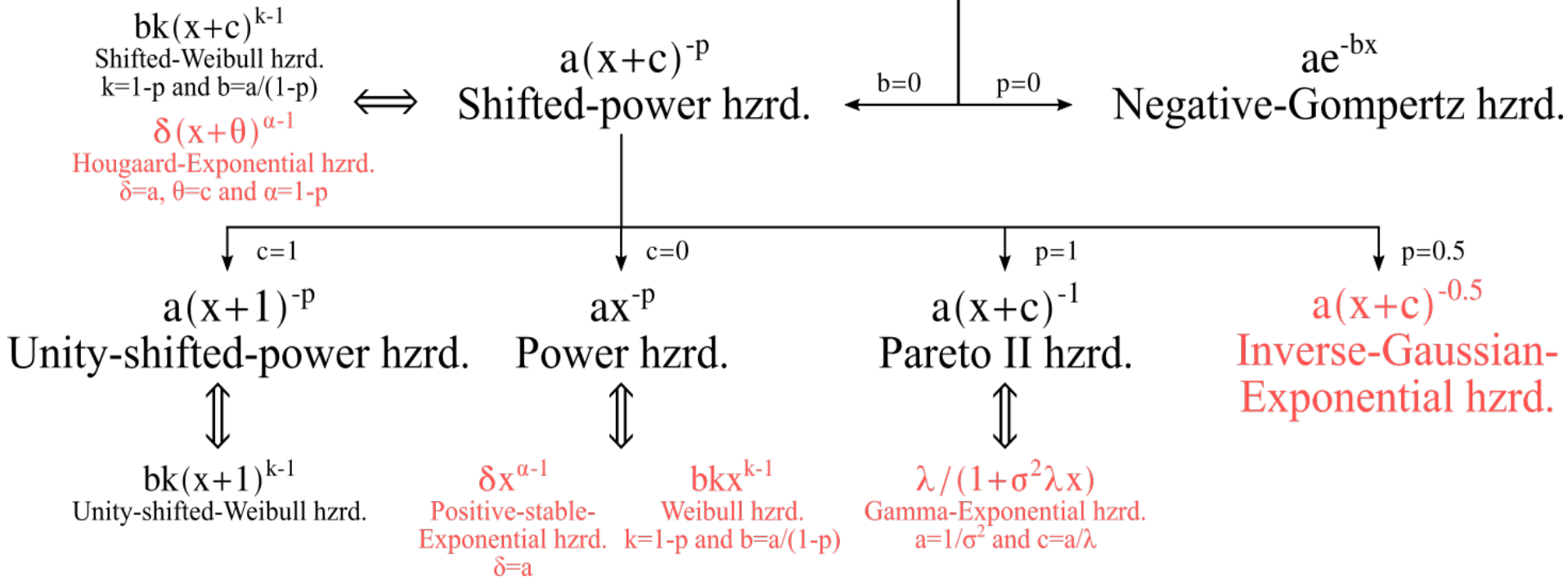
heterogeneous frailty hypothesis

... the frailest individuals die first, causing mean frailty to decline with age.

$$a(x+c)^{-p}e^{-bx}$$

**Power-exponential hzrd.**

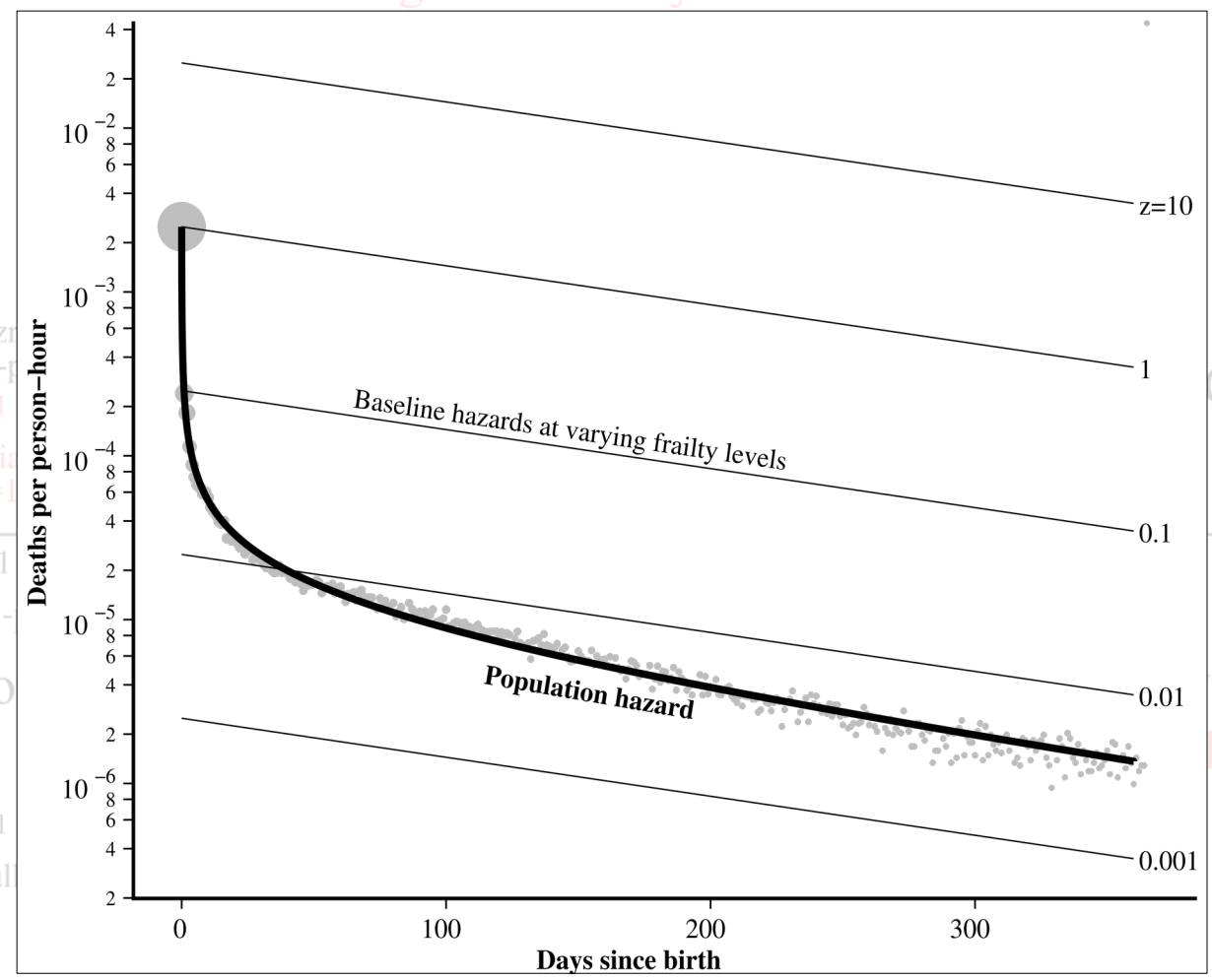
Approximates Hougaard-Gompertz frailty model



# ontogenetic mortality selection

heterogeneous frailty hypothesis

... the frailest individuals die first, causing mean frailty to decline



$bk(x+c)^{k-1}$   
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Hougaard-Exponential  
 $\delta=a, \theta=c$  and  $\alpha=1$

$c=1$

$a(x+1)^{-p}$

Unity-shifted-poisson

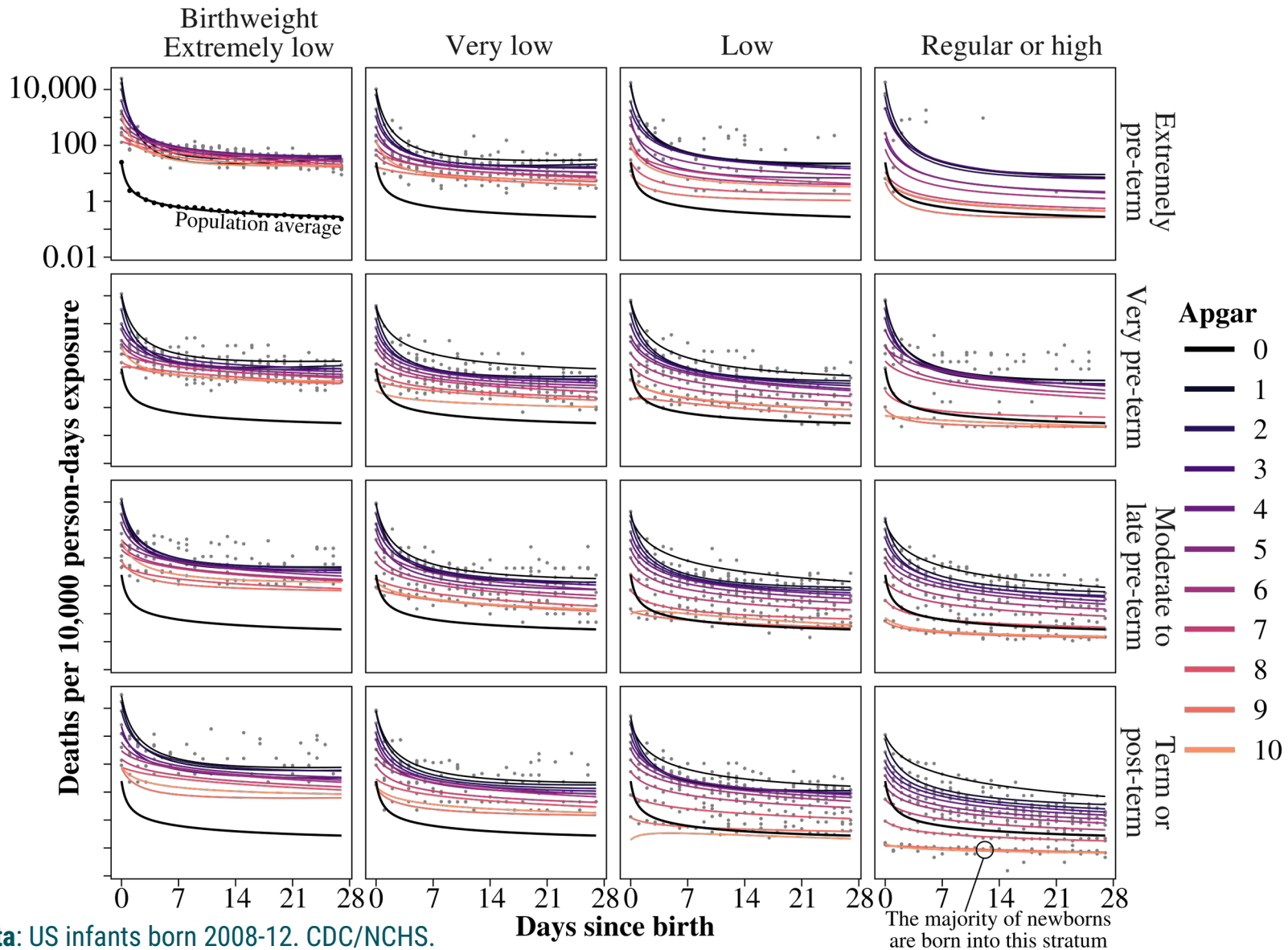


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Unity-shifted-Weibull

$ae^{-bx}$   
Gompertz hzrd

$p=0.5$   
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# “Mortality selection” put to the test

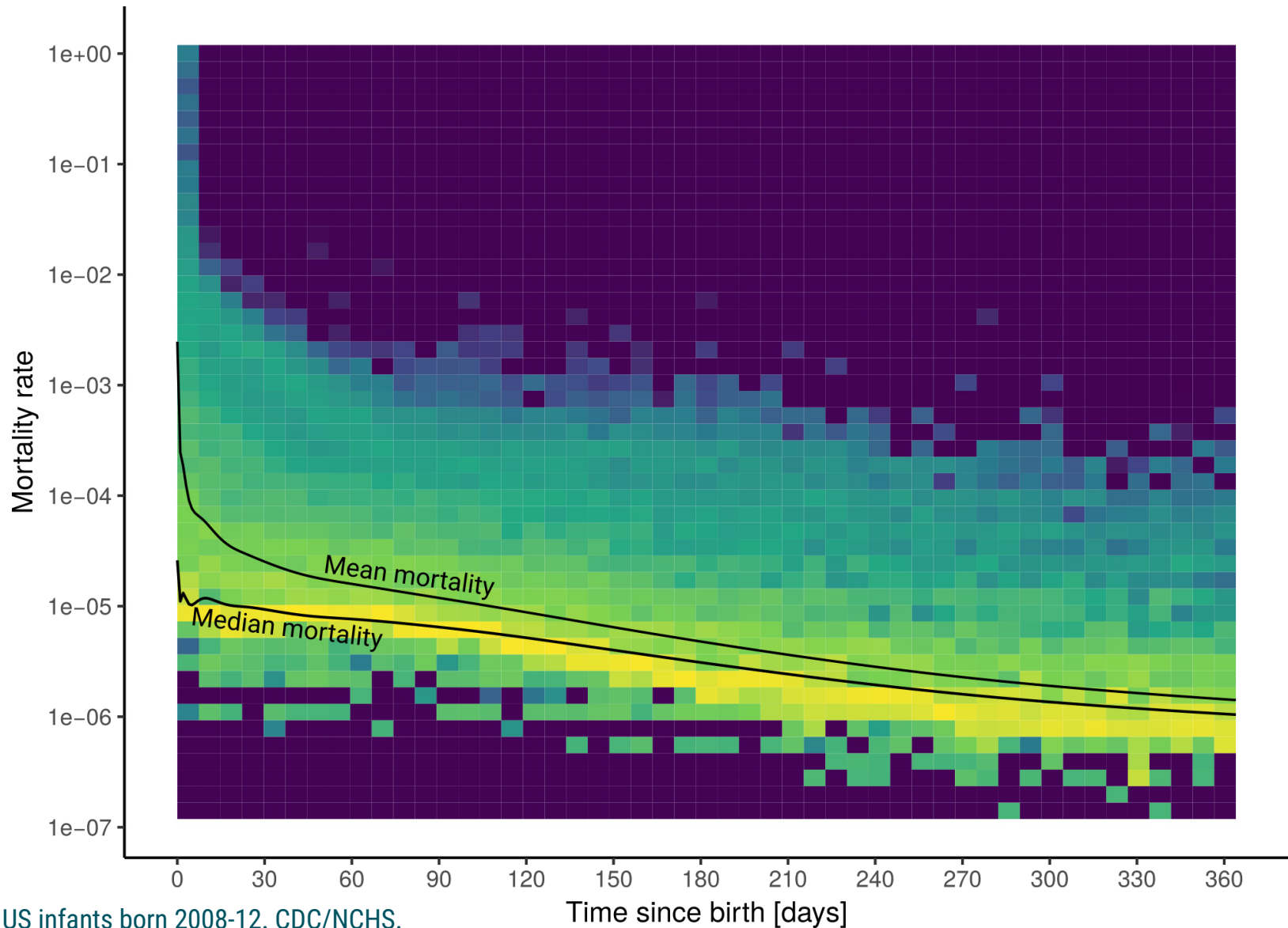


Data: US infants born 2008-12. CDC/NCHS.

Days since birth

The majority of newborns are born into this stratum

# “Mortality selection” put to the test



Data: US infants born 2008-12. CDC/NCHS.

# “Mortality selection” put to the test

## Decomposing change in **average mortality over age**

$$\Delta \bar{m}_j = \underbrace{\sum_k \frac{p_{jk} + p_{j+1,k}}{2} \Delta m_{jk}}_{\text{Direct change } \Delta \bar{m}_j^D} + \underbrace{\sum_k \frac{m_{jk} + m_{j+1,k}}{2} \Delta p_{jk}}_{\text{Compositional change } \Delta \bar{m}_j^C}$$

$$m_{jk} = \frac{D_{jk}}{E_{jk}}$$

$$p_{jk} = \frac{E_{jk}}{\sum_k E_{jk}}$$

$$\bar{m}_j = \frac{\sum_k D_{jk}}{\sum_k E_{jk}}$$

## Decomposing change in **mortality variance over age**

$$\Delta v_j = \underbrace{\sum_k \frac{p_{jk} + p_{j+1,k}}{2} \Delta s_{jk}}_{\text{Direct change } \Delta v_j^D} + \underbrace{\sum_k \frac{s_{jk} + s_{j+1,k}}{2} \Delta p_{jk}}_{\text{Compositional change } \Delta v_j^C}$$

$$v_j(x) = \sum_k p_{jk} s_{jk}, \text{ with } s_{jk} = (m_{jk} - \bar{m}_j)^2$$

## Decomposing change in **mean-mode mortality ratio over age**

$$\Delta r_j = \underbrace{\sum_k \frac{p_{jk} + p_{j+1,k}}{2} \Delta \frac{m_{jk}}{\mathcal{M}_j}}_{\text{Direct change } \Delta r_j^D} + \underbrace{\sum_k \frac{\frac{m_{jk}}{\mathcal{M}_j} + \frac{m_{j+1,k}}{\mathcal{M}_{j+1}}}{2} \Delta p_{jk}}_{\text{Compositional change } \Delta r_j^C}$$

$$\mathcal{M}_j = m_{j,k=r} \text{ with } r \text{ such that } p_{j,k=r} = \max(p_{j1}, \dots, p_{jK})$$

$$\frac{\bar{m}_j}{\mathcal{M}_j} = \sum_k p_{jk} \frac{m_{jk}}{\mathcal{M}_j}$$

# “Mortality selection” put to the test

**Mortality selection along...**

APGAR score ×

Birthweight ×

Gestation at delivery



**... explains**

**21%** of the **mortality decline** over the first day of life, and less than 5% at later ages

**... explains**

**23%** of the **relative rate of decline in the hazard of death** at birth, and less than 5% at later ages

**... explains**

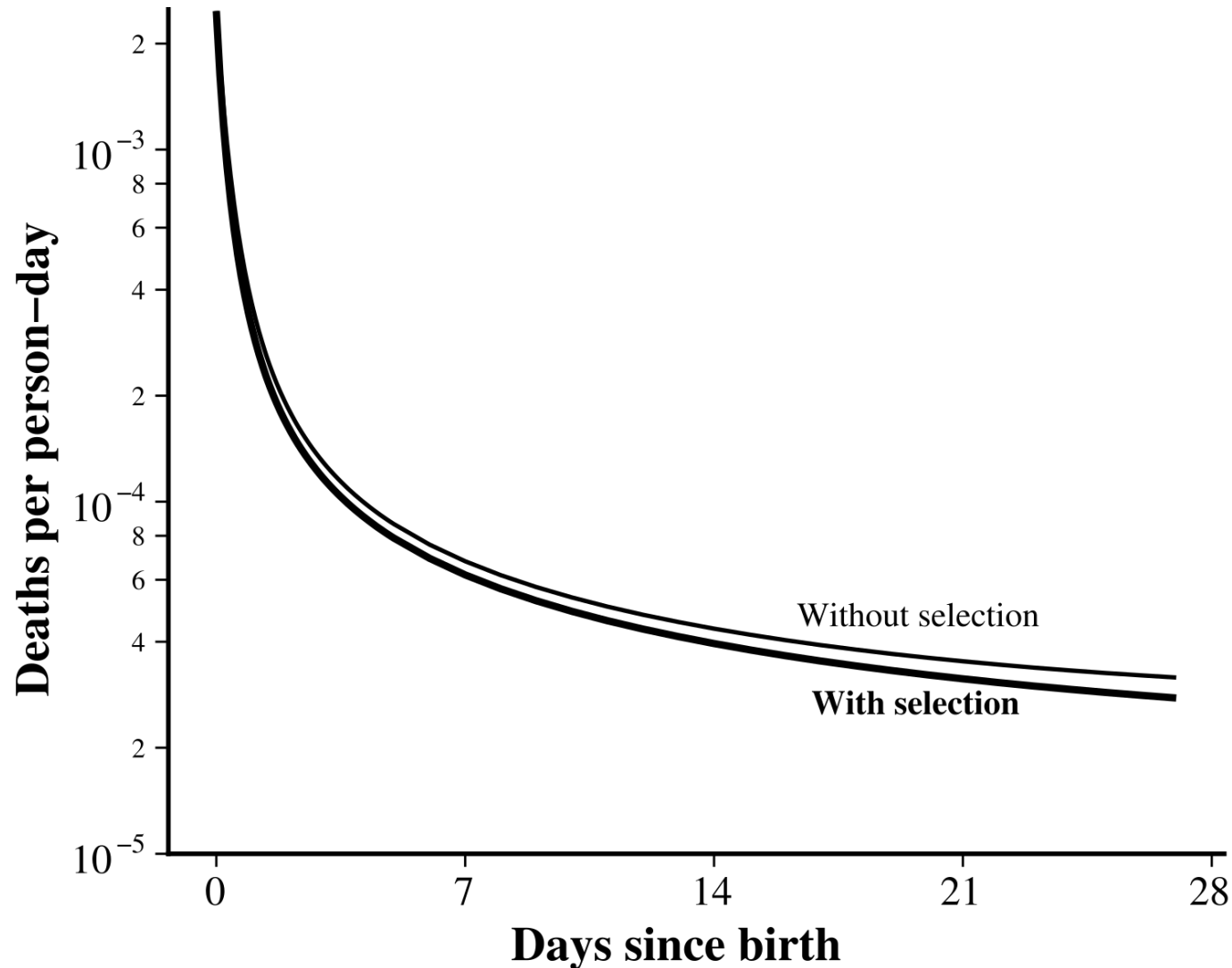
**21%** of the **decline in the variance of mortality risks** over the first day of life, and less than 5% at later ages

**... explains**

**23%** of the **decline in the positive skewness of mortality risks** over the first day of life, and less than 5% at later ages

**Data:** US infants born 2008-12. CDC/NCHS.

# “Mortality selection” put to the test



Data: US infants born 2008-12. CDC/NCHS.

# **The transitional shock of birth**

# The transitional shock of birth

“**Ontogenescence** is a population-level phenomenon in which the **death rate** of each cohort tends to **decrease with increasing age** between conception and maturity.”  
(Levitis 2011)

Levitis, Daniel A. (2011). “Before senescence: the evolutionary demography of ontogenesis”. In: *Proceedings of the Royal Society B* 278, pp. 801–809.

# The transitional shock of birth

802 D. A. Levitis *Review. Mortality before senescence*

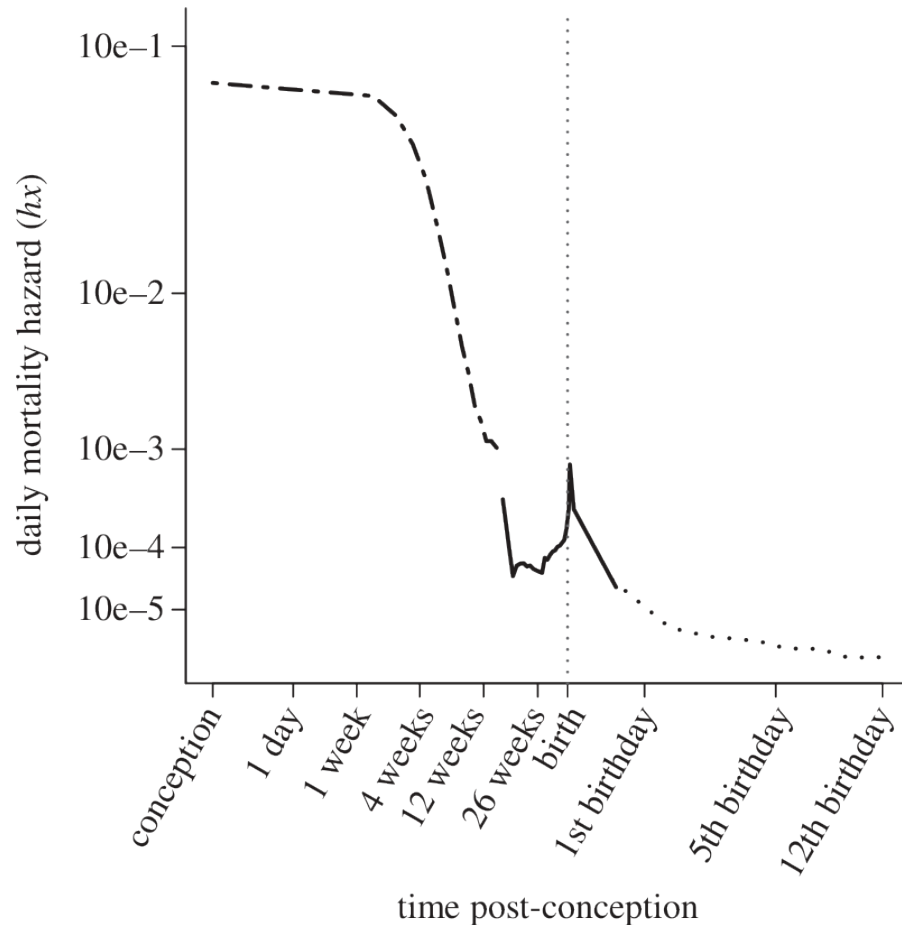


Figure 1. Mortality hazard from conception to 12th birthday.

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# The transitional shock of birth

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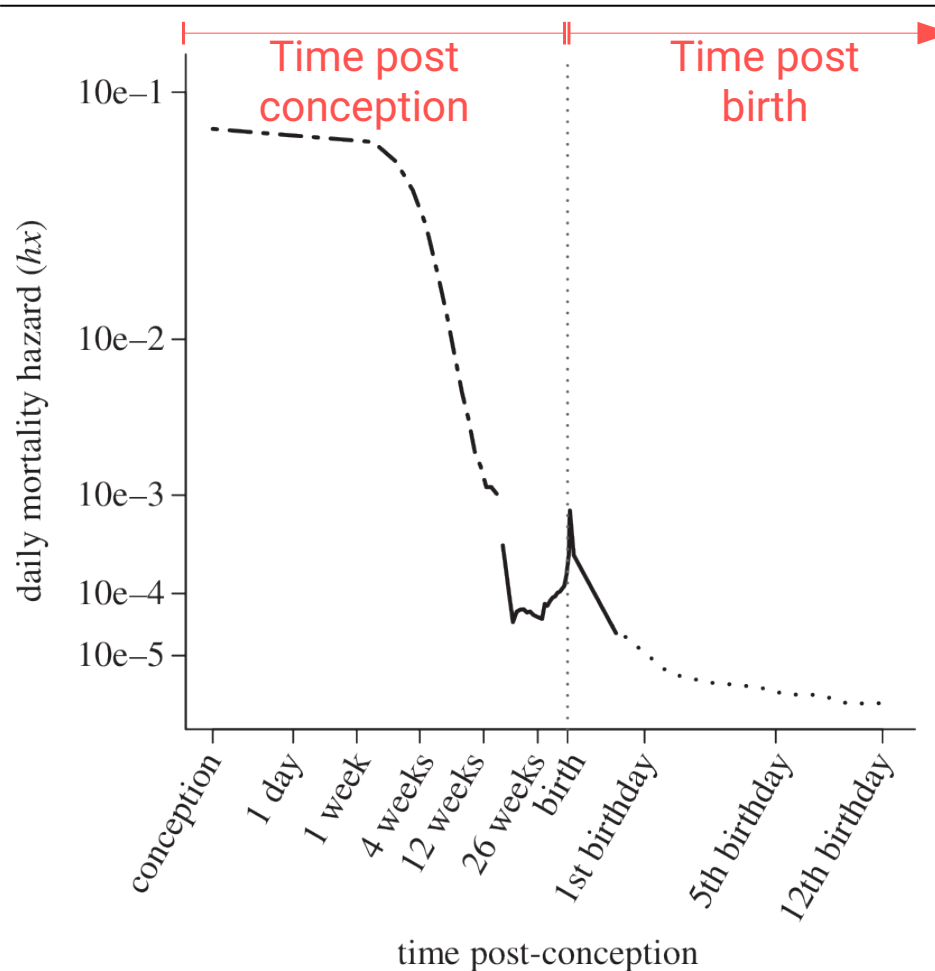
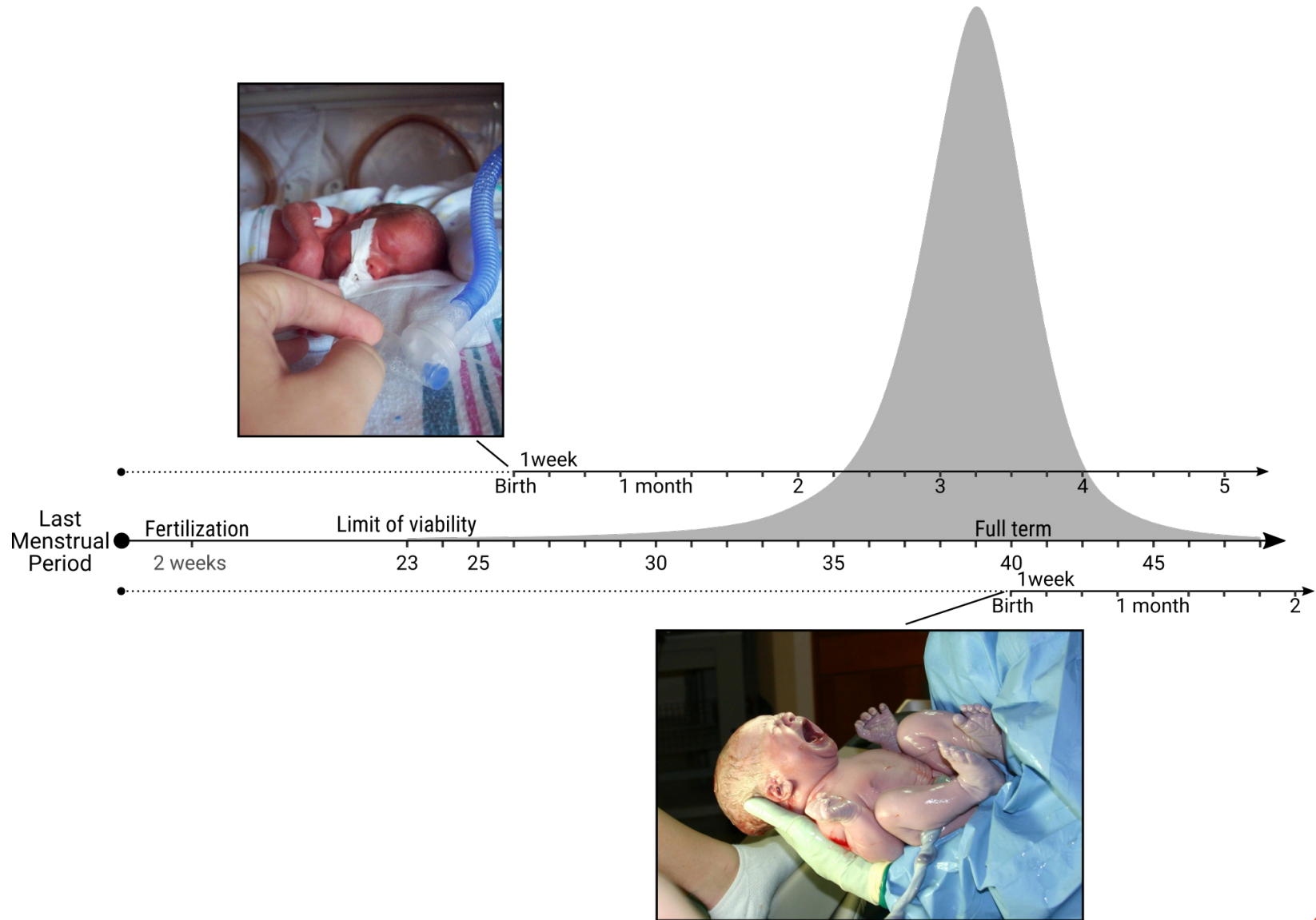


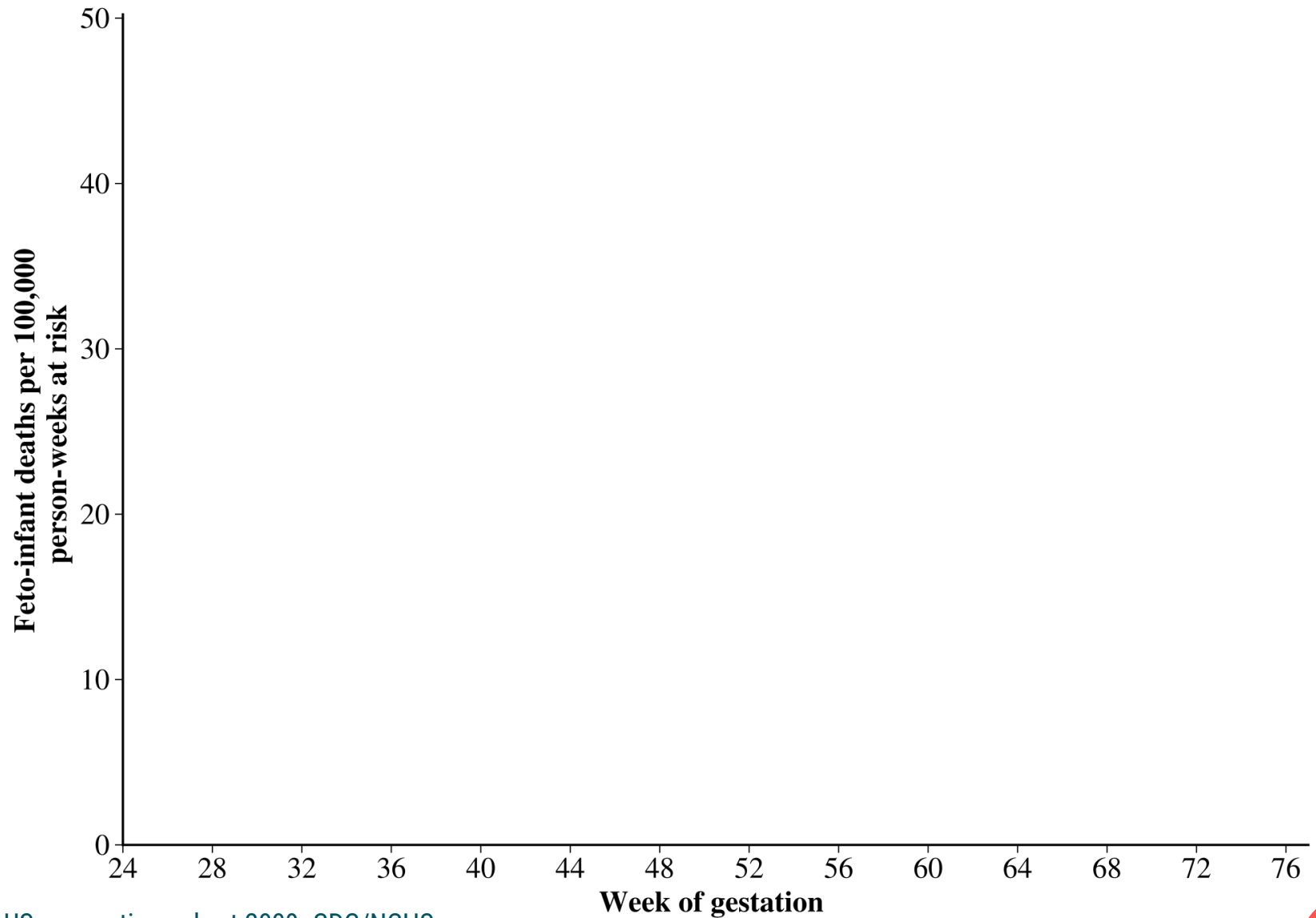
Figure 1. Mortality hazard from conception to 12th birthday.

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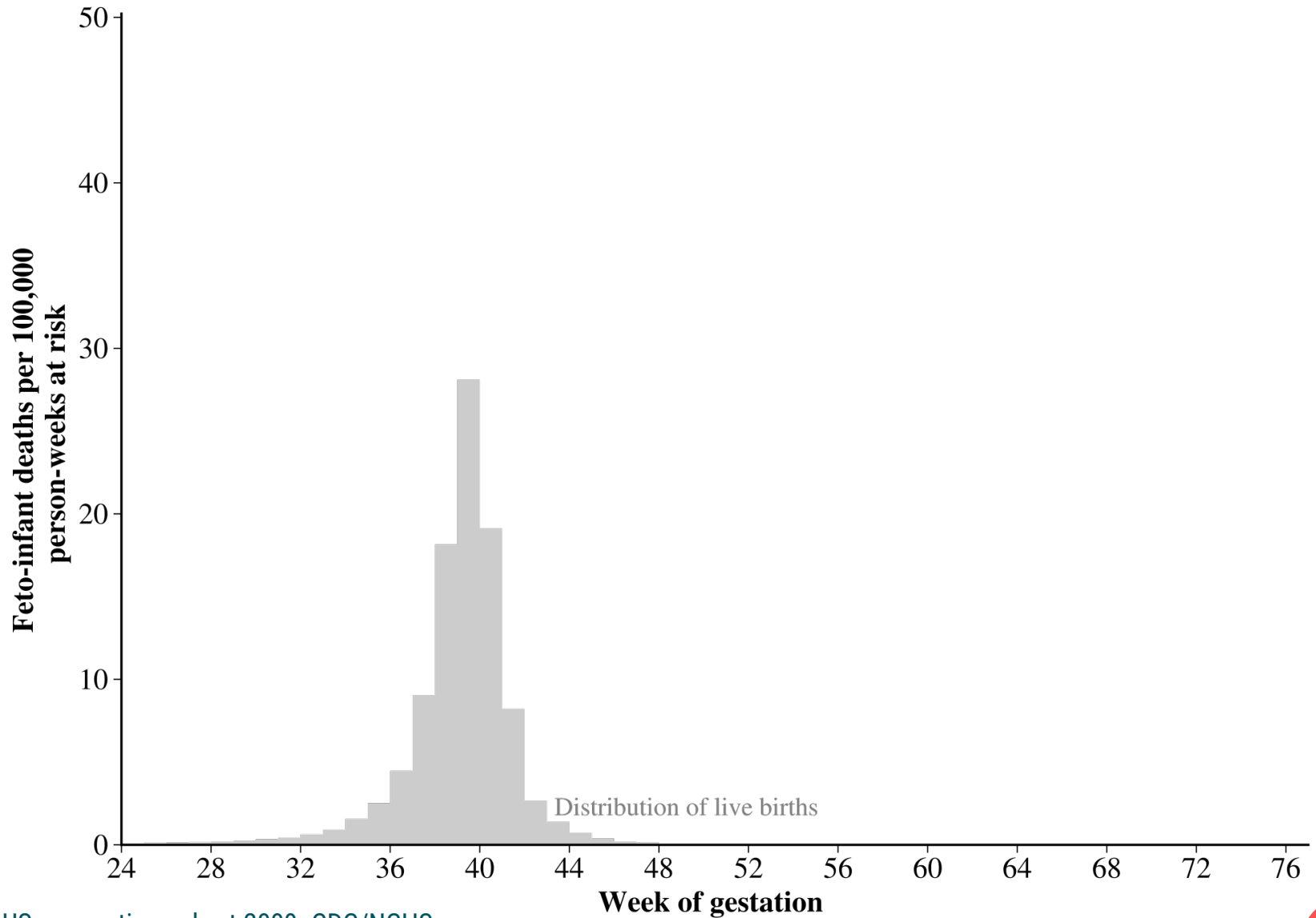


# The transitional shock of birth



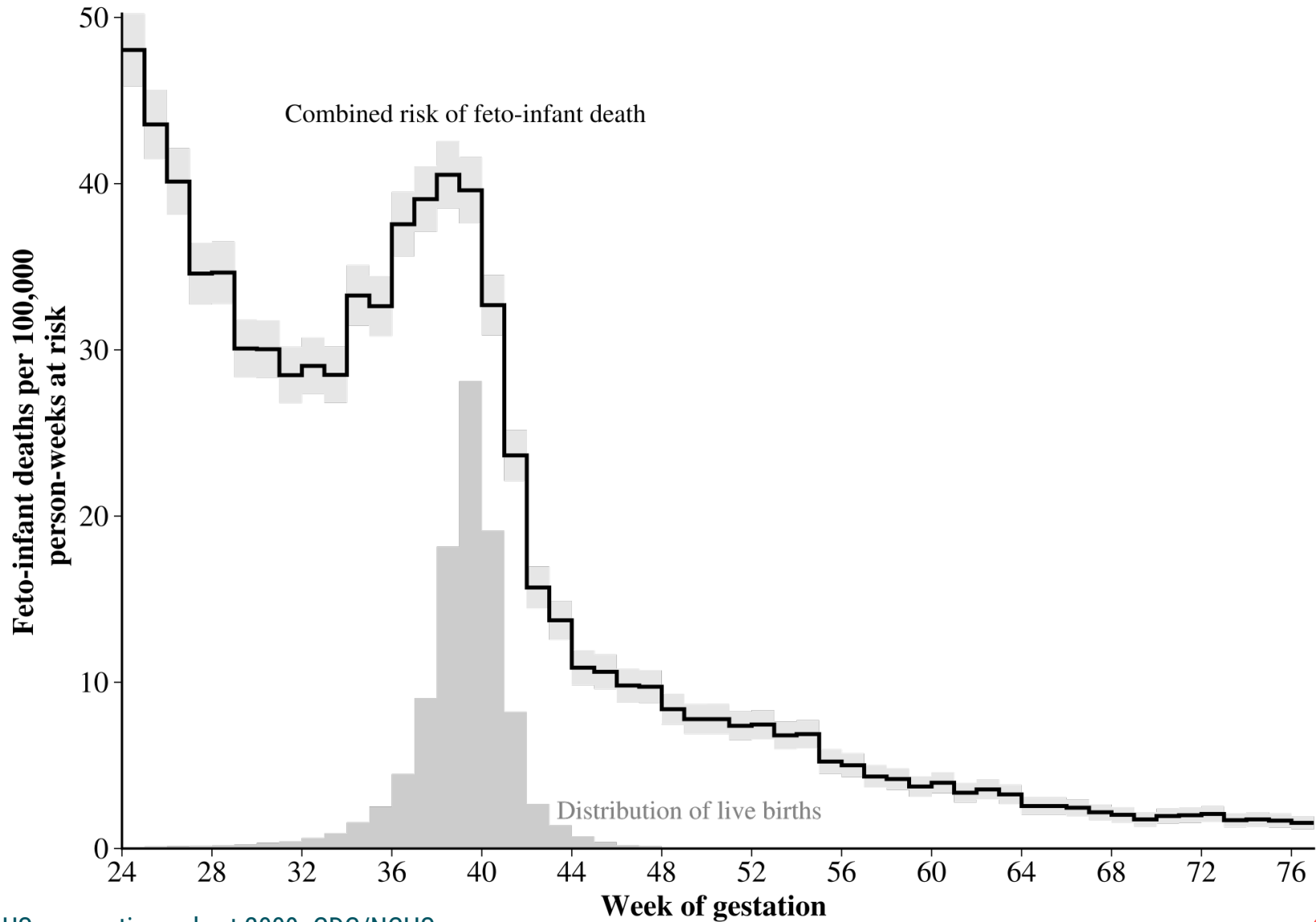
**Data:** US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth



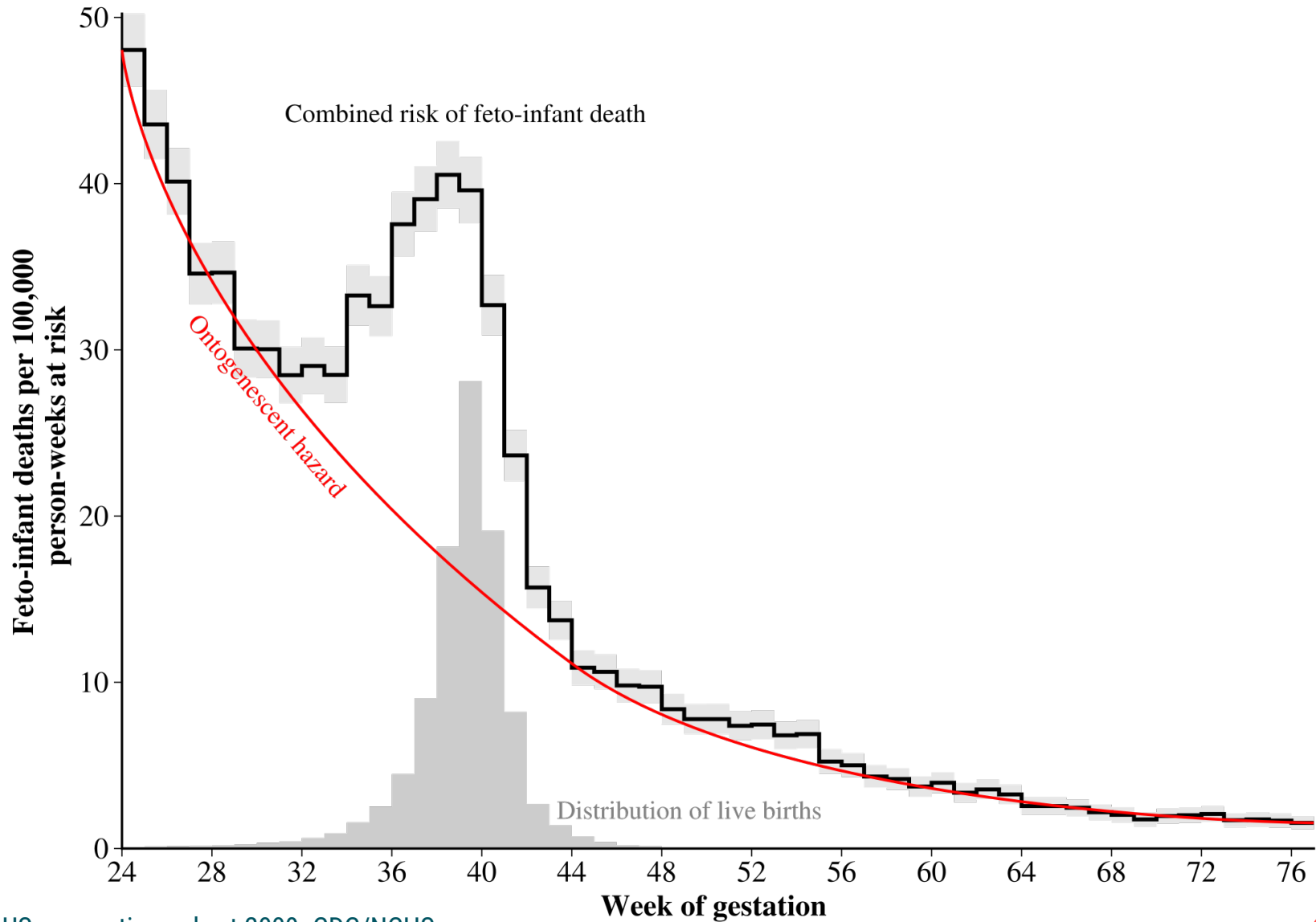
Data: US conception cohort 2009. CDC/NCHS.

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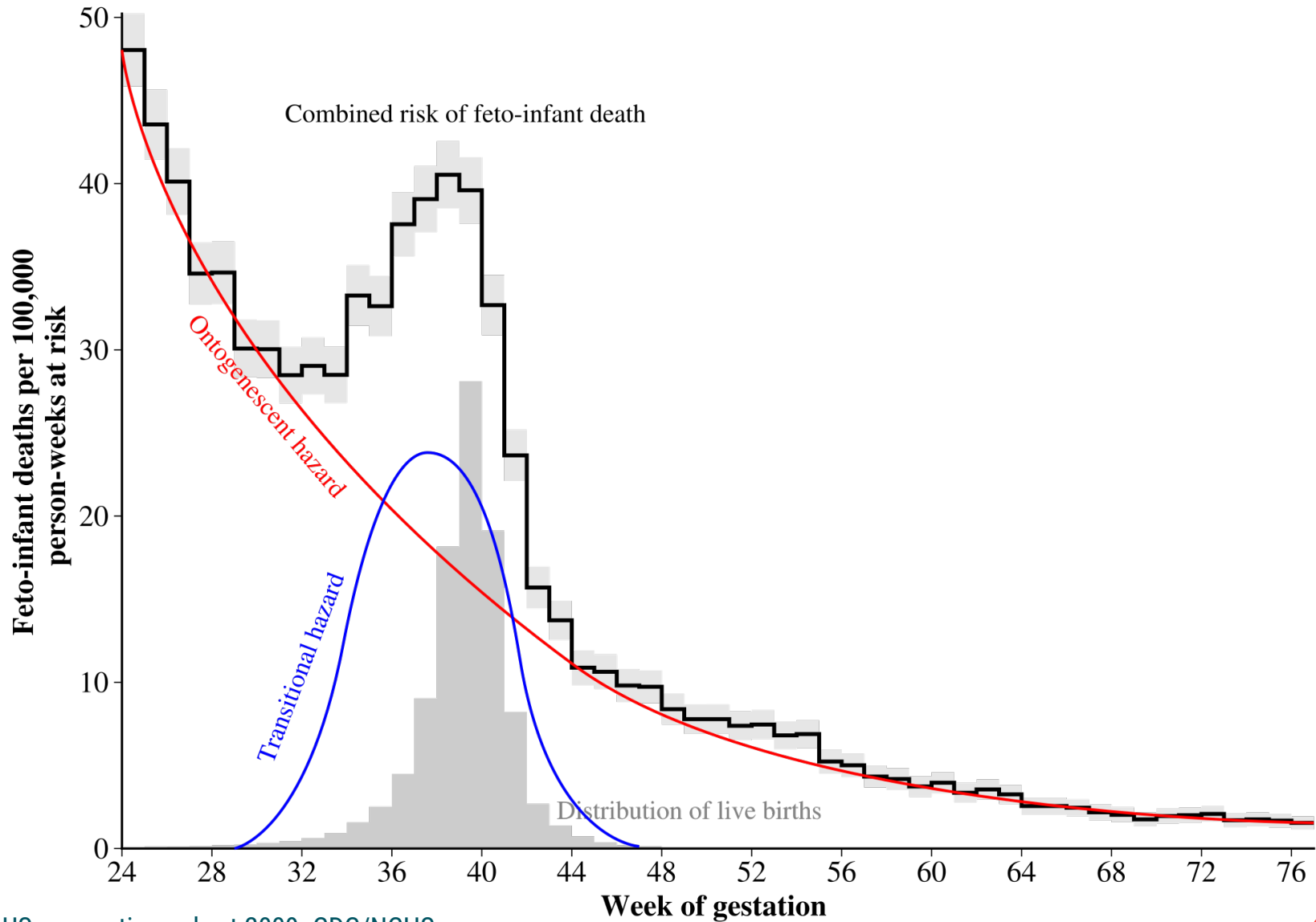
Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth



Data: US conception cohort 2009. CDC/NCHS.

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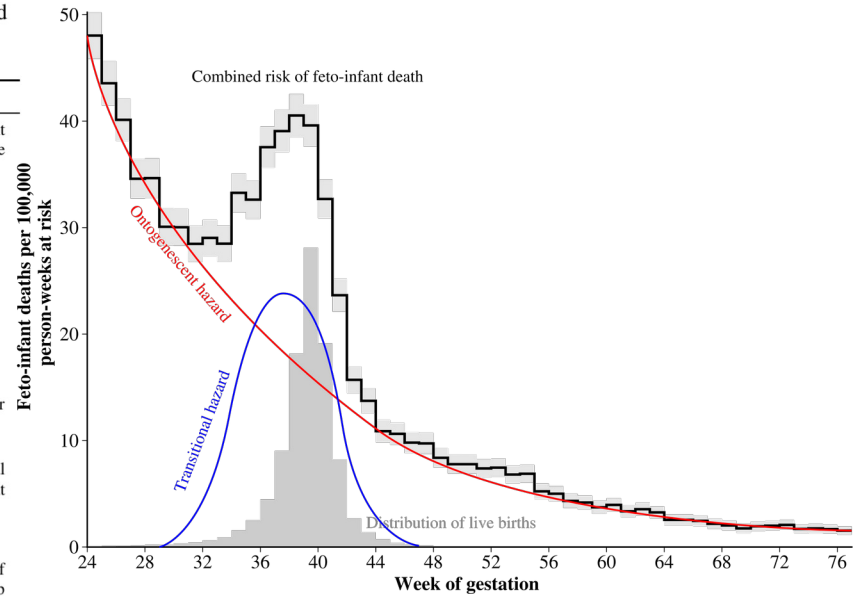


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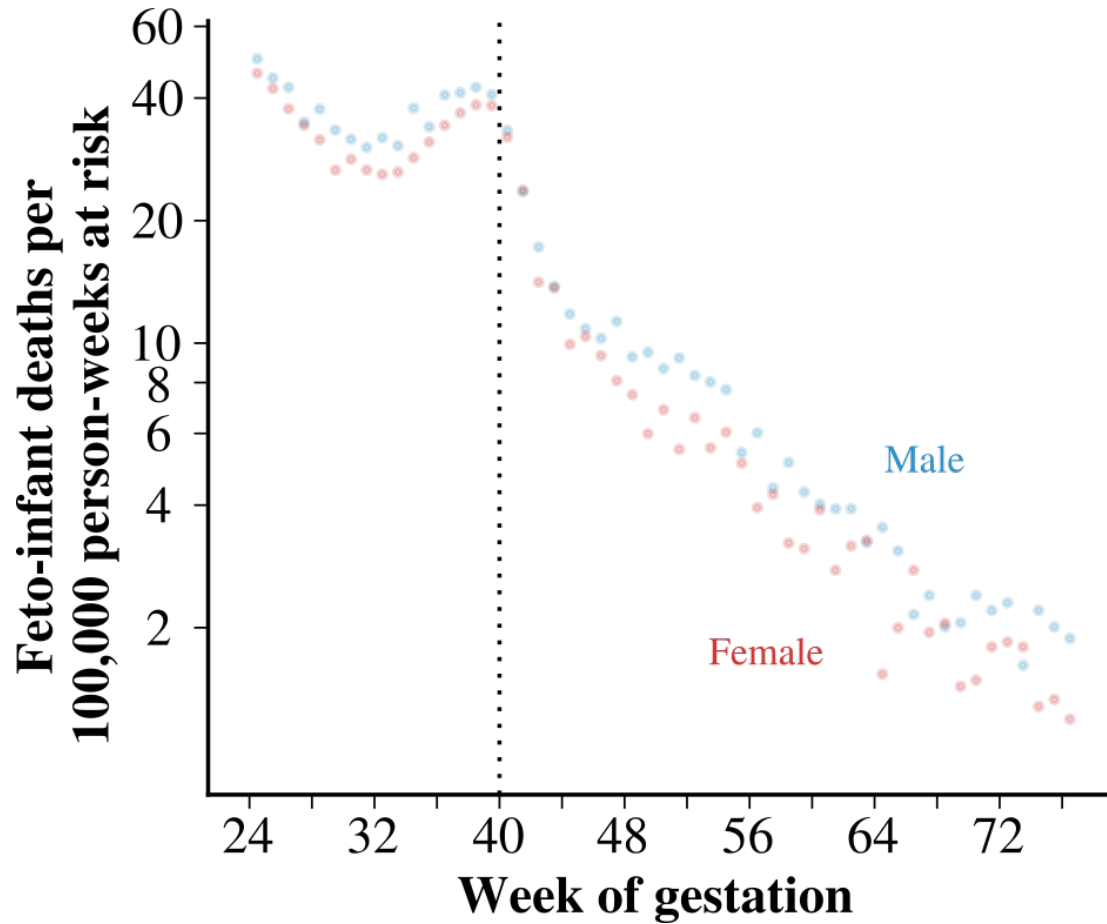
# The transitional shock of birth

**Table 1:** Parametric specification of the feto-infant mortality trajectory over age of gestation and derived quantities.

Ontogenescent component	Transitional component
<p><i>Ontogenescent hazard</i> The instantaneous risk of fetal or infant death at gestational age <math>t = x + 24</math> due to causes with a continuously declining incidence.</p> $h^O(x) = a_1 \exp(-bx)$ <p><i>Cumulative ontogenescent hazard</i></p> $H^O(x) = \int_0^x h^O(s) ds = \frac{a_1 - a_1 \exp(-bx)}{b}$ <p><math>a_1</math> <i>Level of feto-infant mortality</i> The approximate hazard of feto-infant death at age of fetal viability.</p> <p><math>b</math> <i>Rate of ontogenescence</i> The relative rate of feto-infant mortality decline over gestational age in absence of birth hump.</p>	<p><i>Transitional hazard</i> The instantaneous risk of fetal or infant death at gestational age <math>x</math> due to causes associated with the timing of onset of labor.</p> $h^T(x) = a_2 \exp\left(-\frac{(x-c)^2}{2\sigma^2}\right)$ <p><i>Cumulative transitional hazard</i></p> $H^T(x) = \int_0^x h^T(s) ds = a_2 \sigma \sqrt{\pi/2} [\text{erf}(A) + \text{erf}(B)],$ <p>where <math>A = \frac{c}{\sqrt{2}\sigma}</math>, <math>B = \frac{x-c}{\sqrt{2}\sigma}</math>, and <math>\text{erf}(\cdot)</math> is the Gaussian error function.</p> <p><math>a_2</math> <i>Magnitude of birth hump</i> The instantaneous risk of fetal or infant death contributed by the birth-hump component at its peak.</p> <p><math>c</math> <i>Location of birth hump</i> The gestational age <math>t = c + 24</math> coinciding with the peak of the risk of fetal or infant death contributed by the birth-hump component.</p> <p><math>\sigma</math> <i>Spread of transitional shock</i> The curvature of the risk of feto-infant death around its peak. Higher values flatten the birth hump.</p>
<b>Combined hazard</b>	
<p><i>Hazard of feto-infant death</i> The instantaneous risk of fetal or infant death <math>x</math> weeks past fetal viability.</p> $h(x) = h^O(x) + h^T(x)$ <p><i>Feto-infant survival curve</i> The probability of surviving <math>x</math> weeks past fetal-viability.</p> $S(x) = \exp\left(-H^O(x) - H^T(x)\right)$ <p><i>Cumulative incidence of feto-infant death</i> Probability of fetal or infant death <math>x</math> weeks past fetal-viability.</p> $F(x) = 1 - S(x)$	
<b>Competing risks inference</b>	
<p>Cumulative incidence of feto-infant death due to causes associated with the timing of onset of labor.</p> $F^T(x) = \int_0^x S(s)h^T(s) ds$ <p>Share of feto-infant deaths over <math>x</math> weeks following fetal viability contributed by the "birth hump".</p> $\rho(x) = \frac{F^T(x)}{F(x)}$	

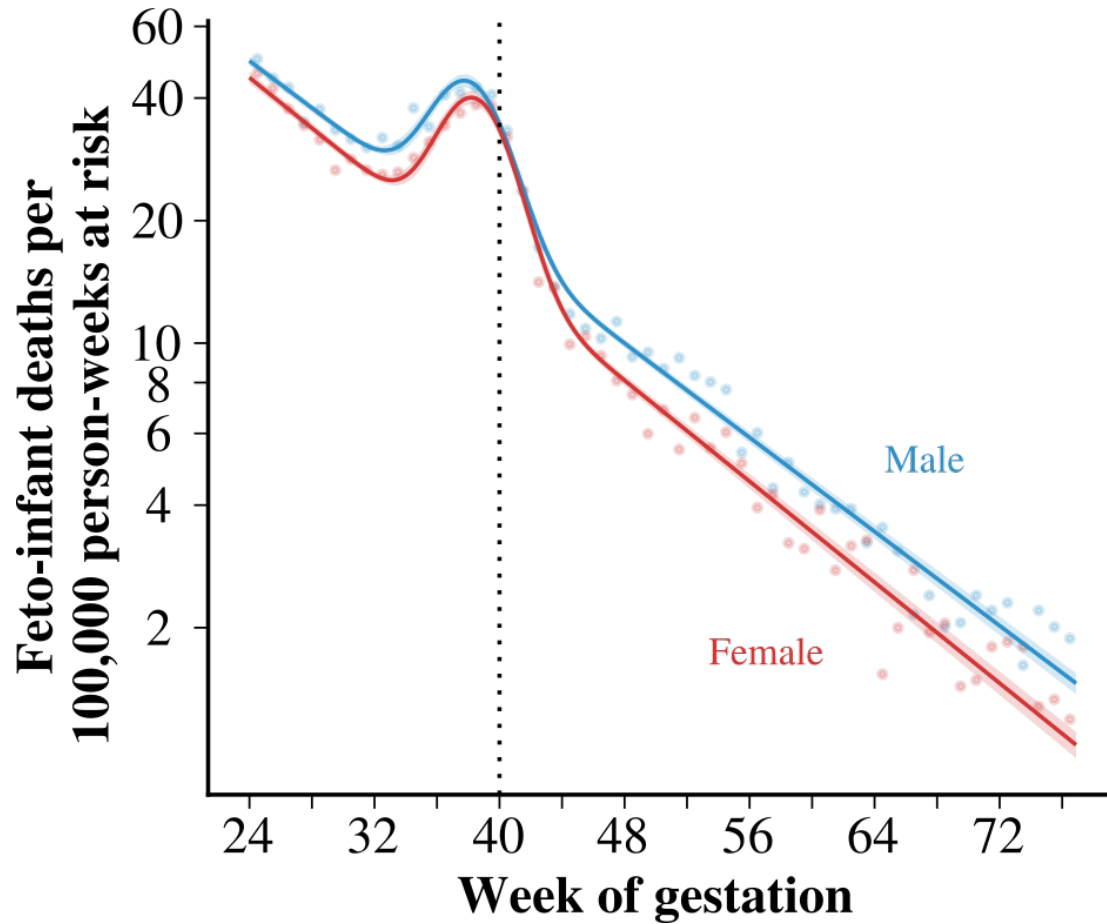


# The transitional shock of birth



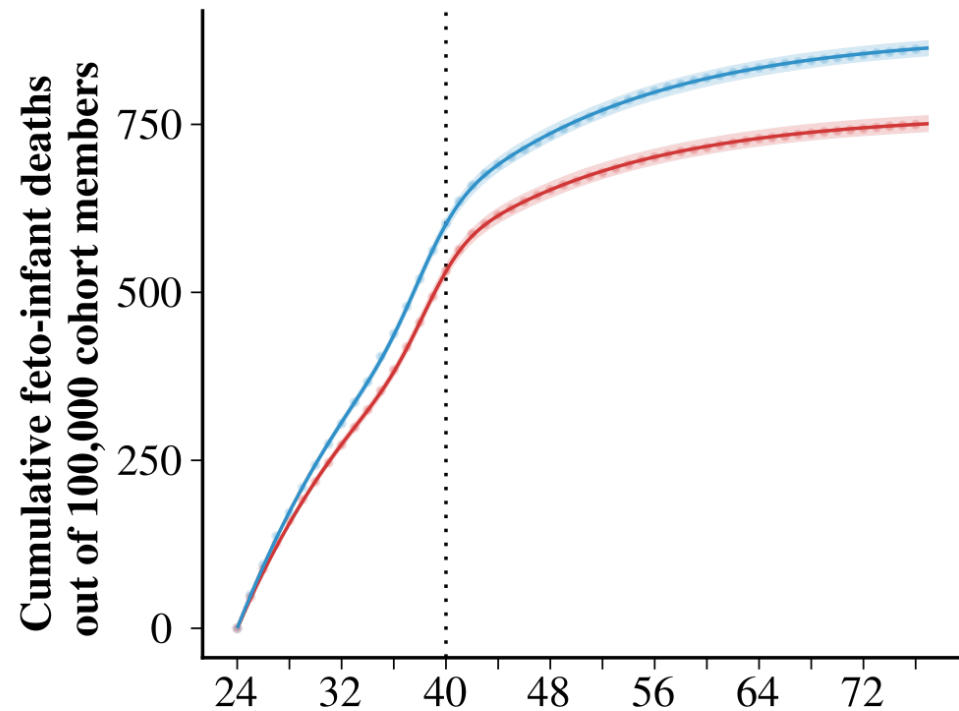
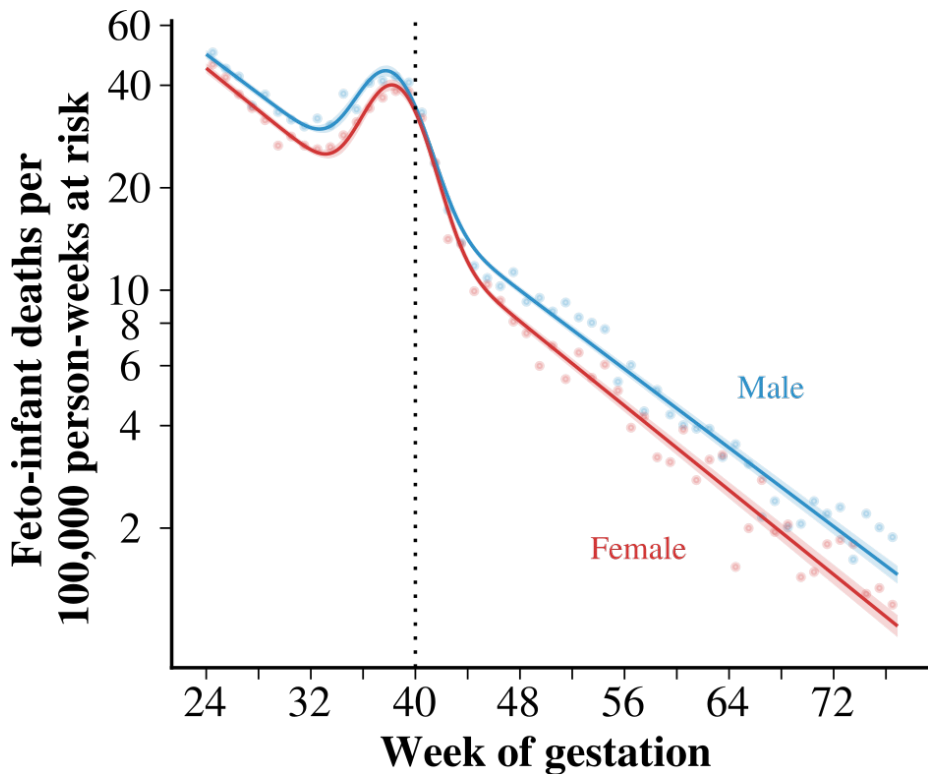
Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth



Data: US conception cohort 2009. CDC/NCHS.

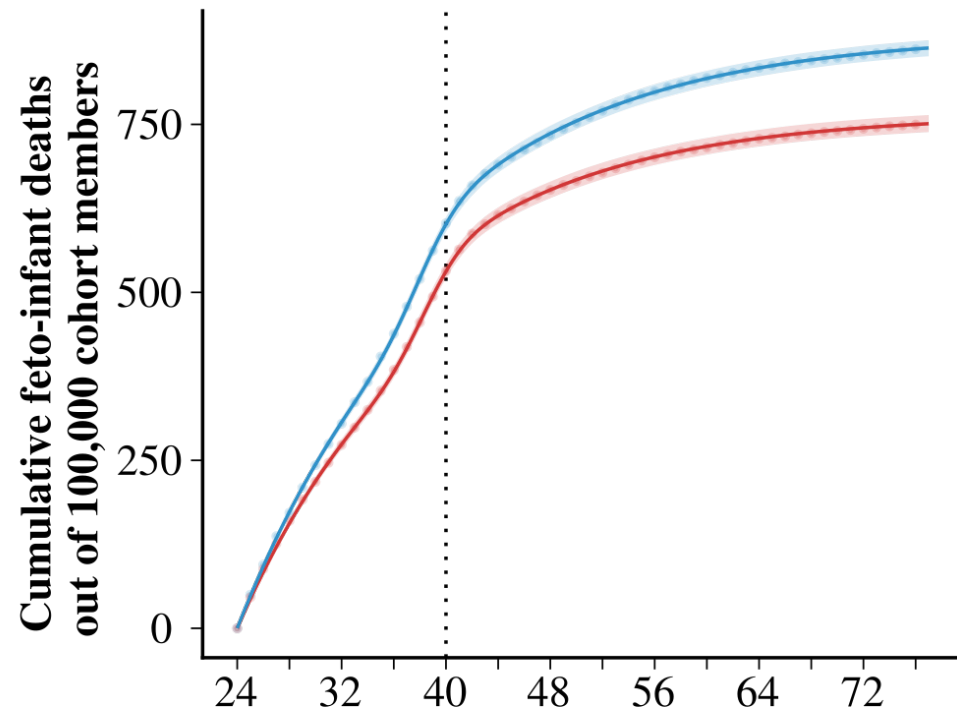
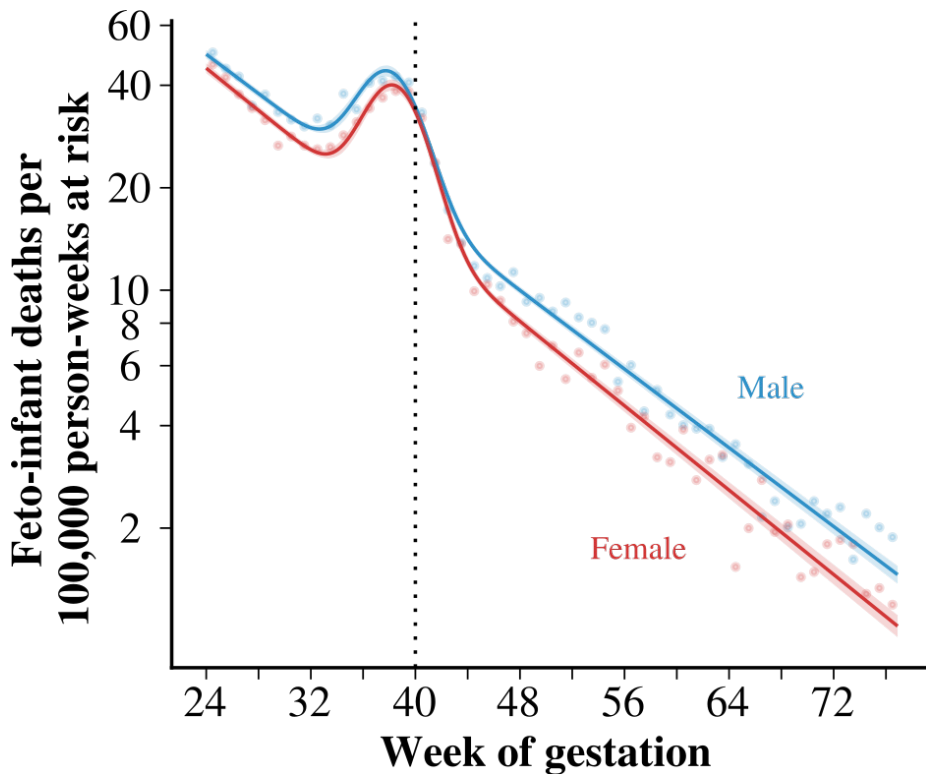
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# The transitional shock of birth

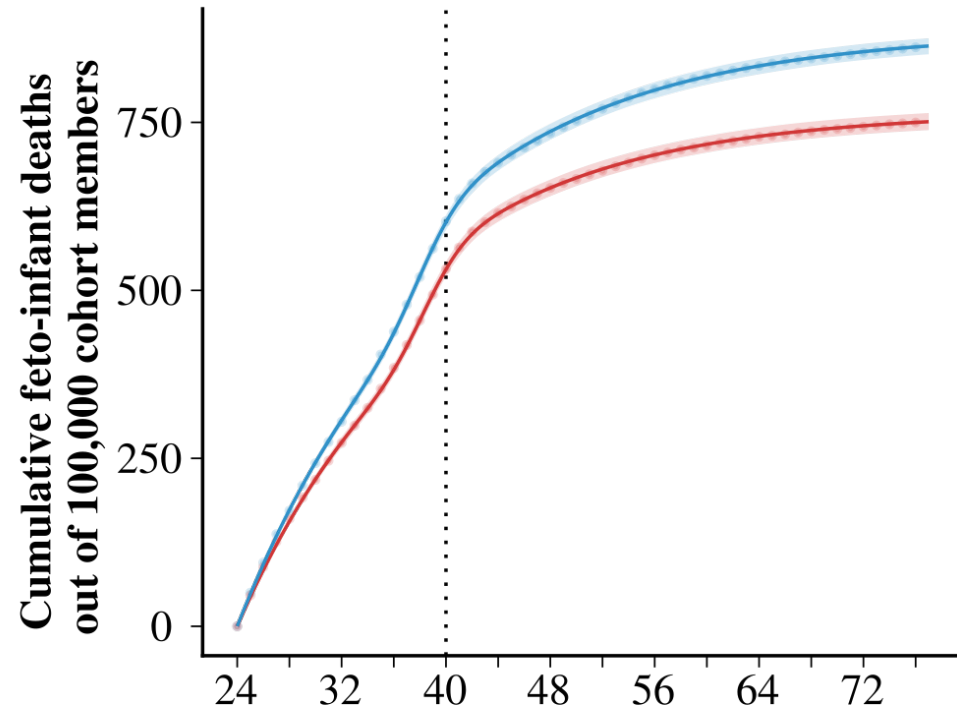
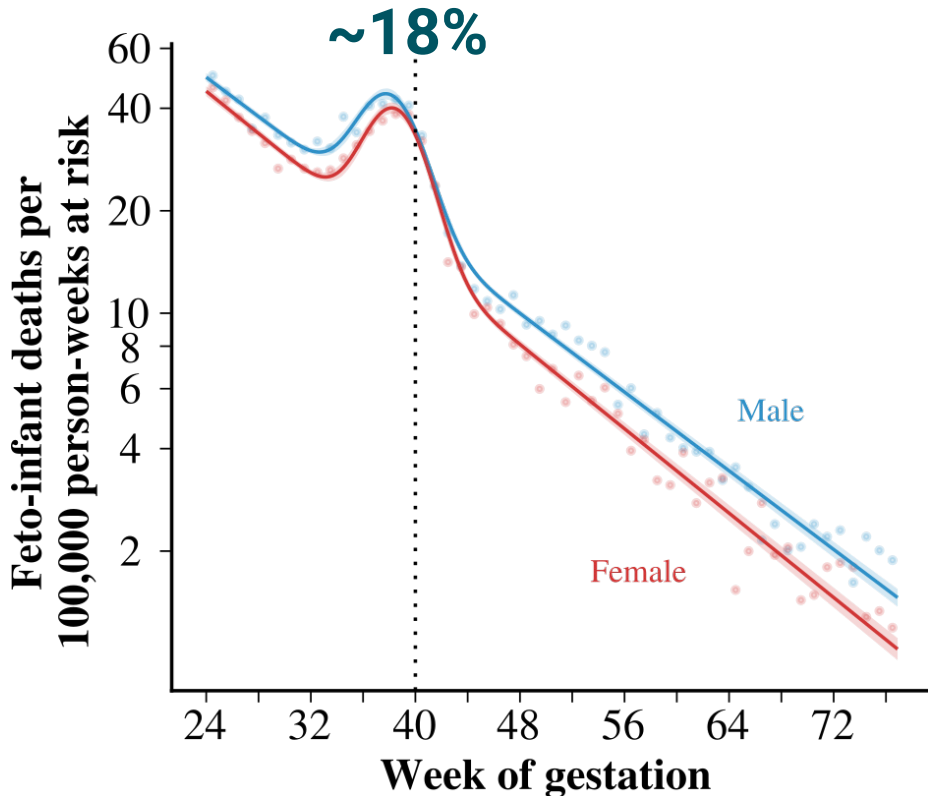
Share of 1-year post-viability deaths due to “birth hump”:



Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth

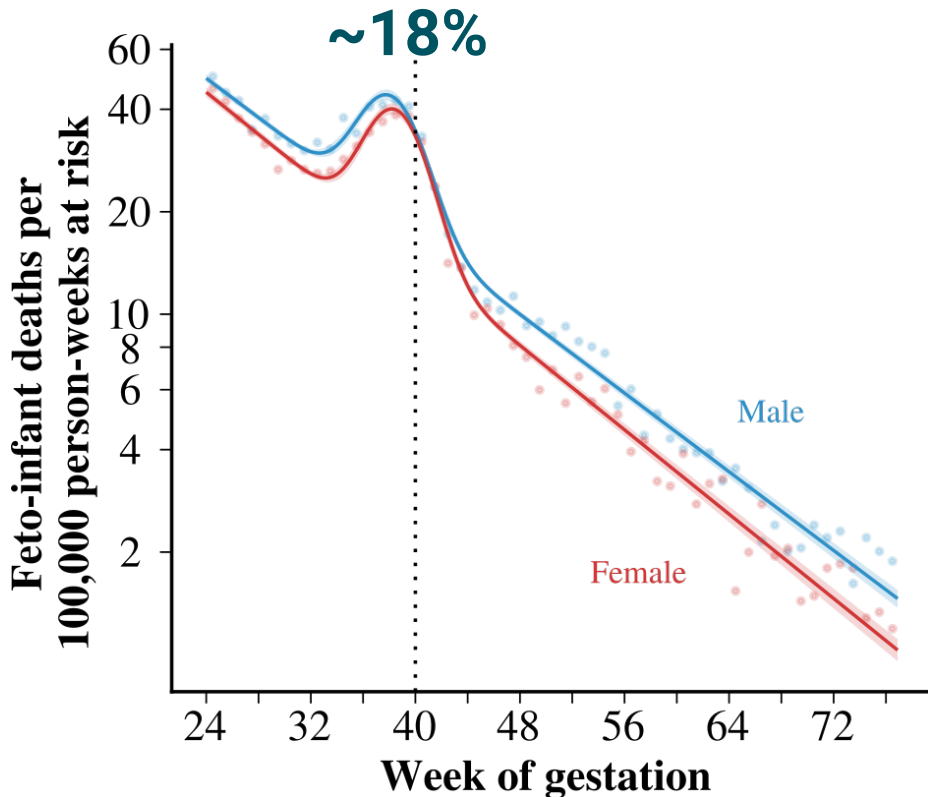
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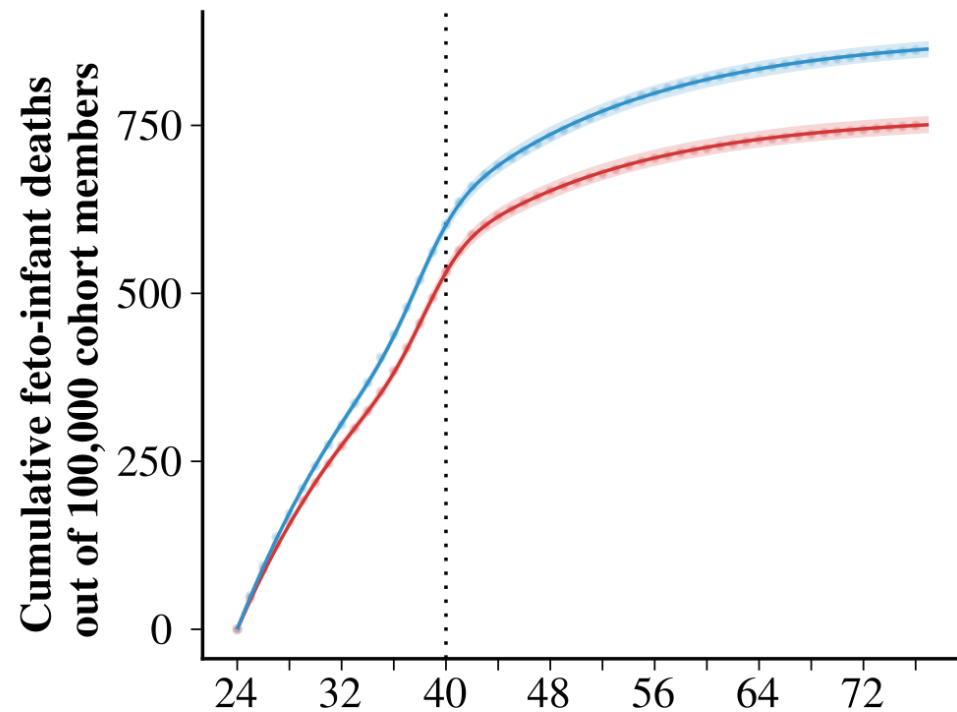
Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth

Share of 1-year post-viability deaths due to “birth hump”:



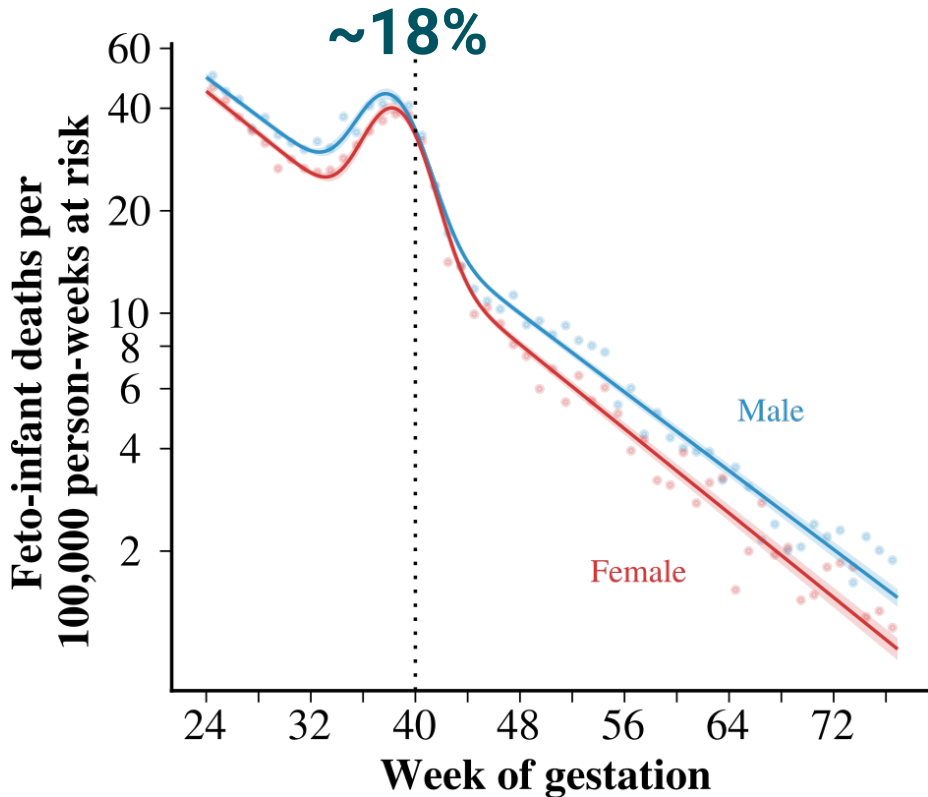
1-year odds of post-viability death:



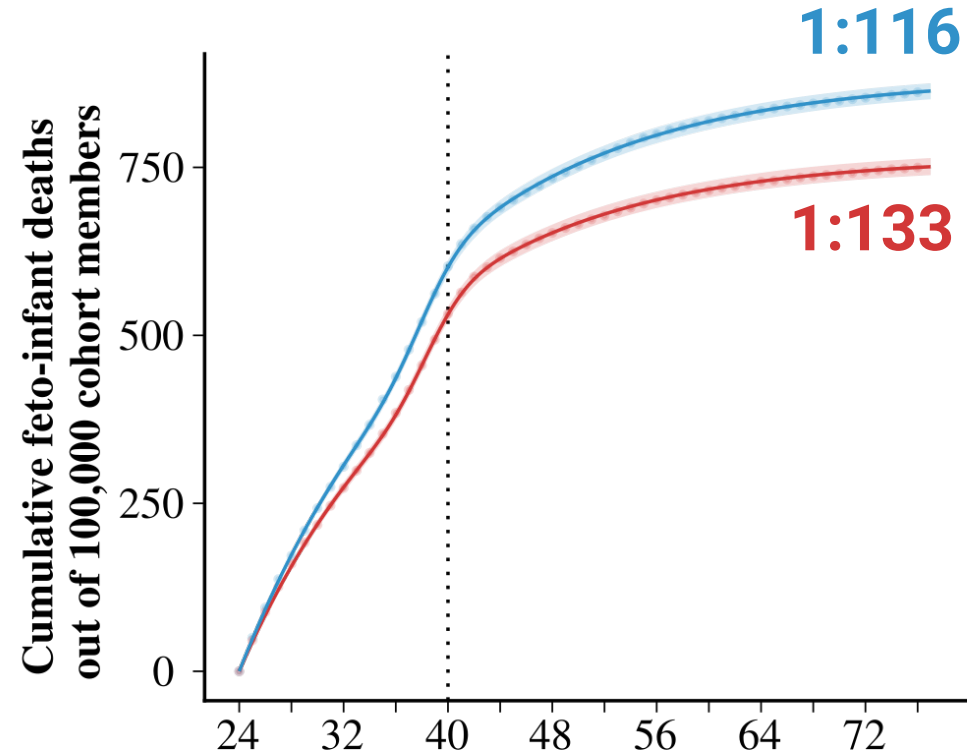
Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth

Share of 1-year post-viability deaths due to “birth hump”:

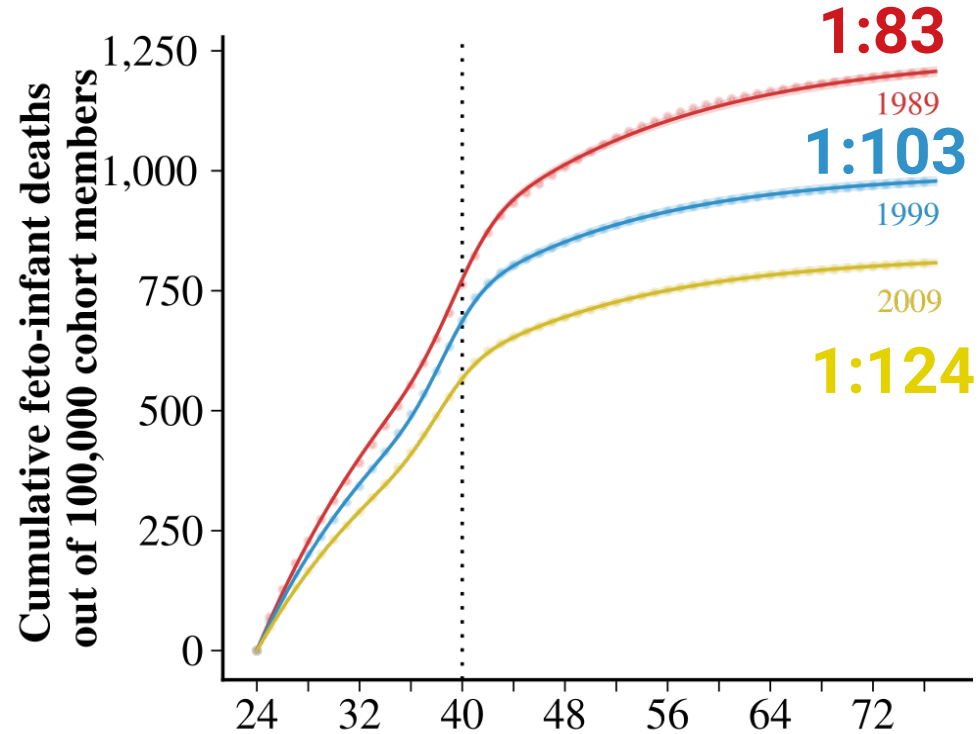
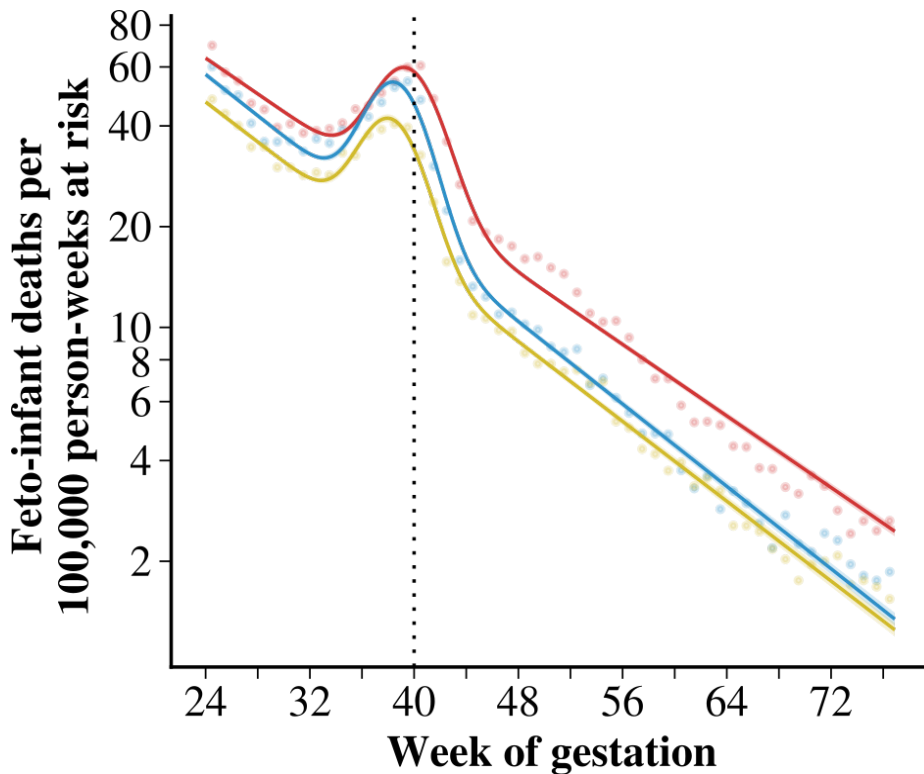


1-year odds of post-viability death:



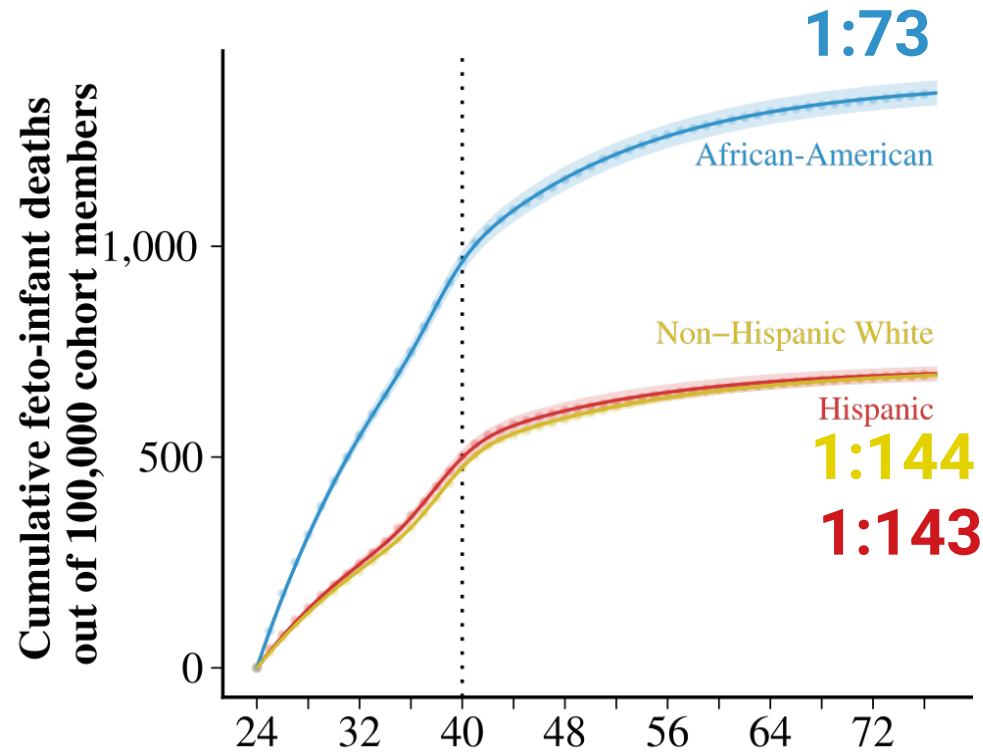
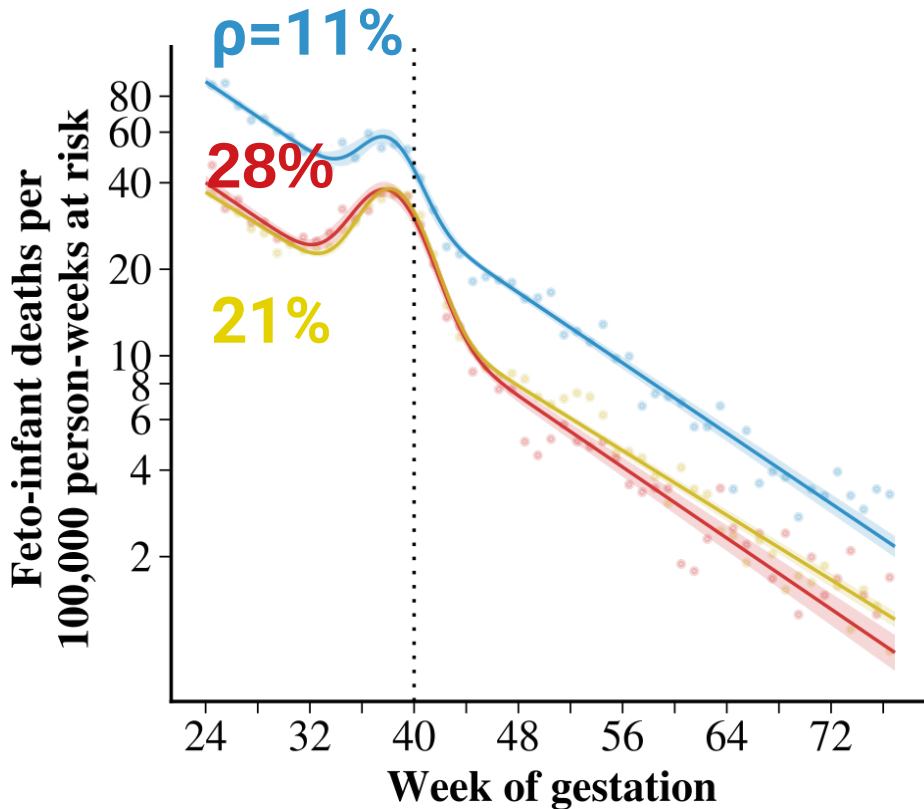
Data: US conception cohort 2009. CDC/NCHS.

# The transitional shock of birth



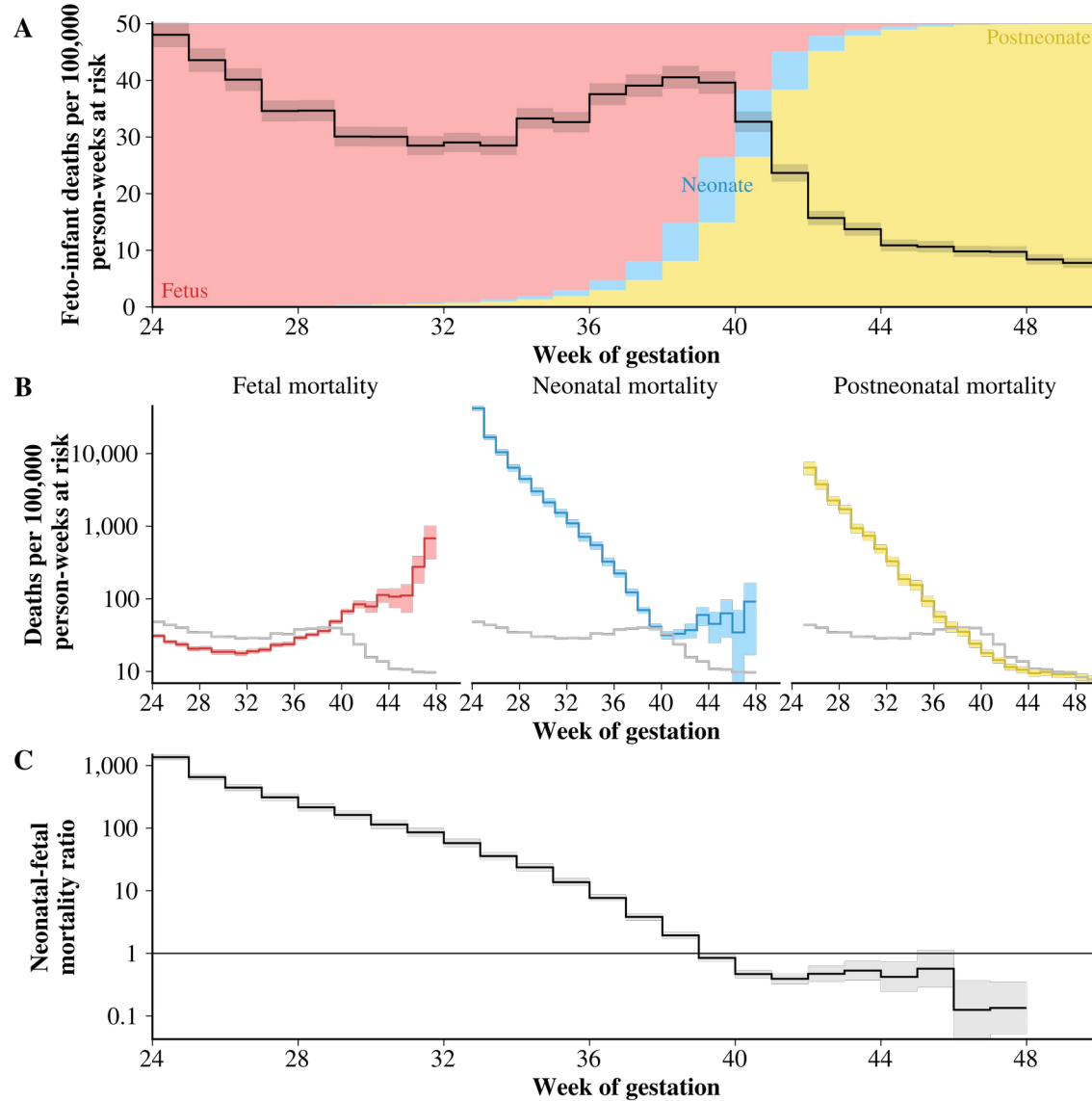
Data: US conception cohorts 1989, 1999, 2009. CDC/NCHS.

# The transitional shock of birth



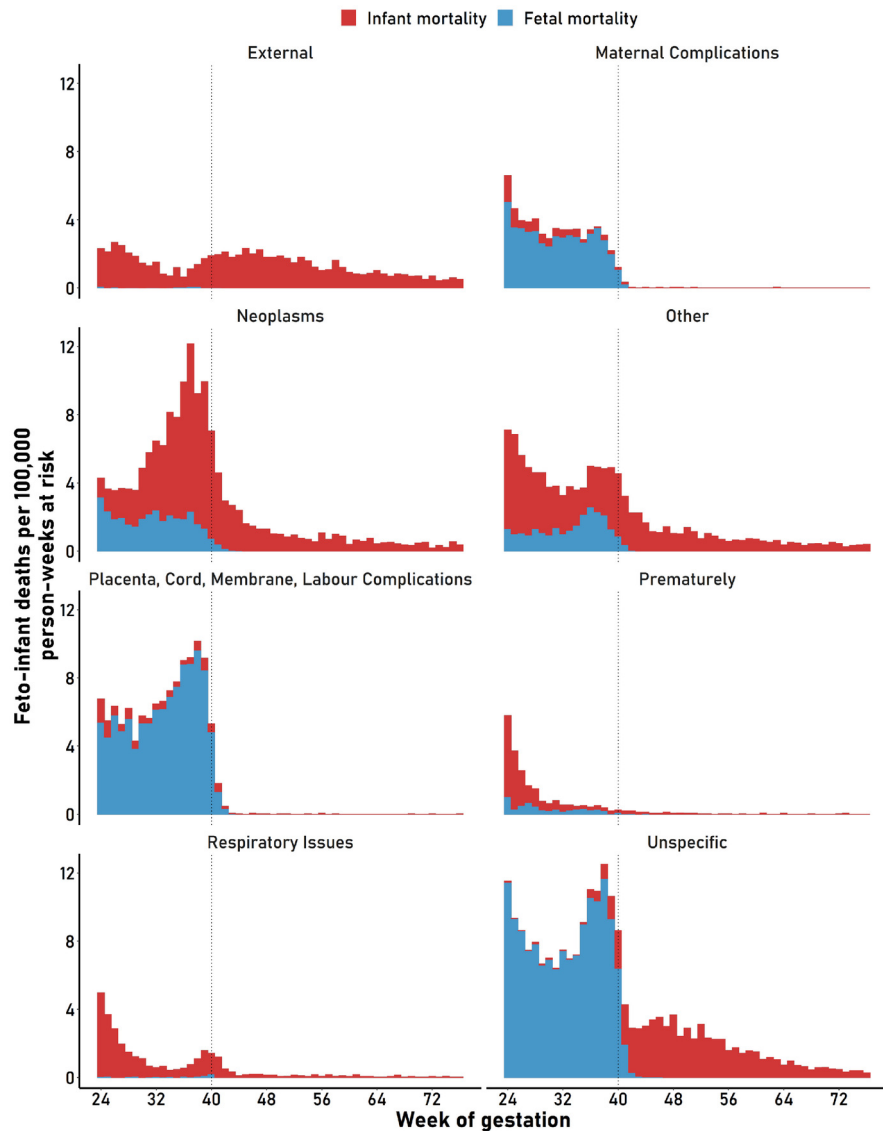
Data: US conception cohorts 1989, 1999, 2009. CDC/NCHS.

# Transitional dynamics of the “birth hump”



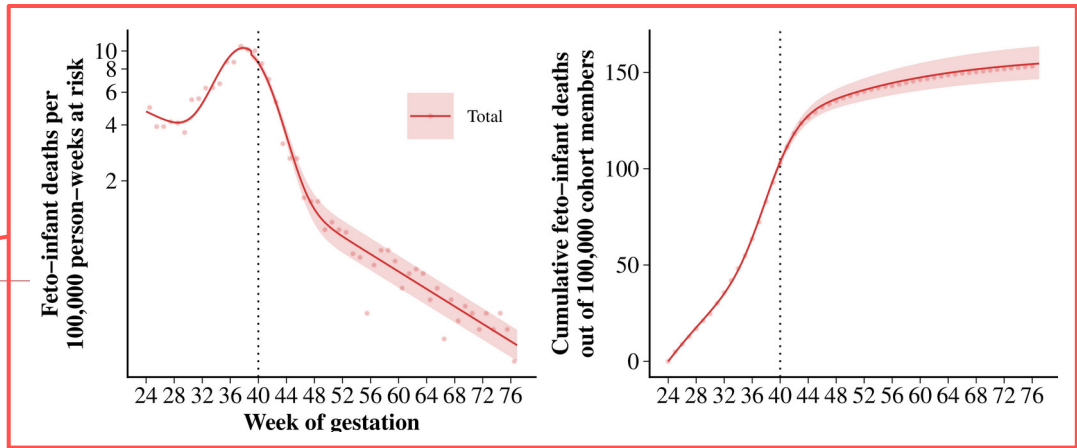
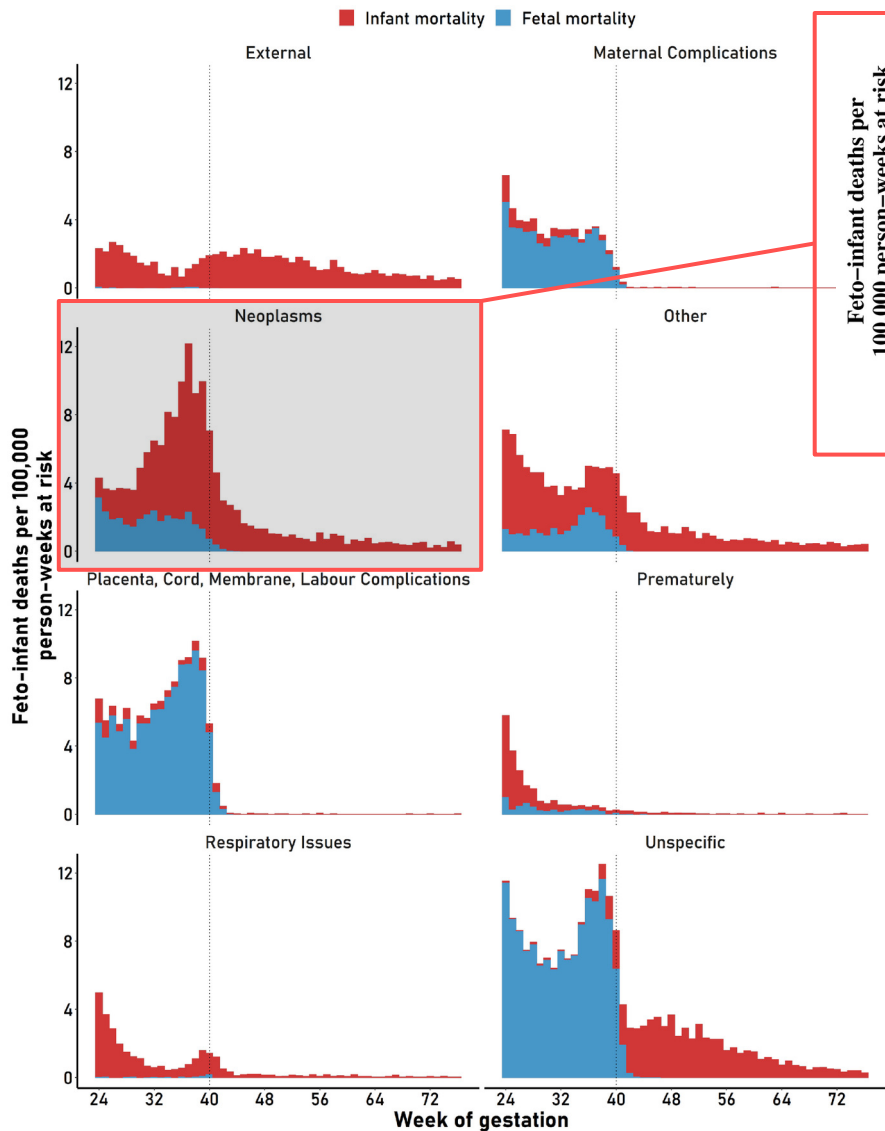
Data: US conception cohorts 1989, 1999, 2009. CDC/NCHS.

# Cause of death composition of the “birth hump”



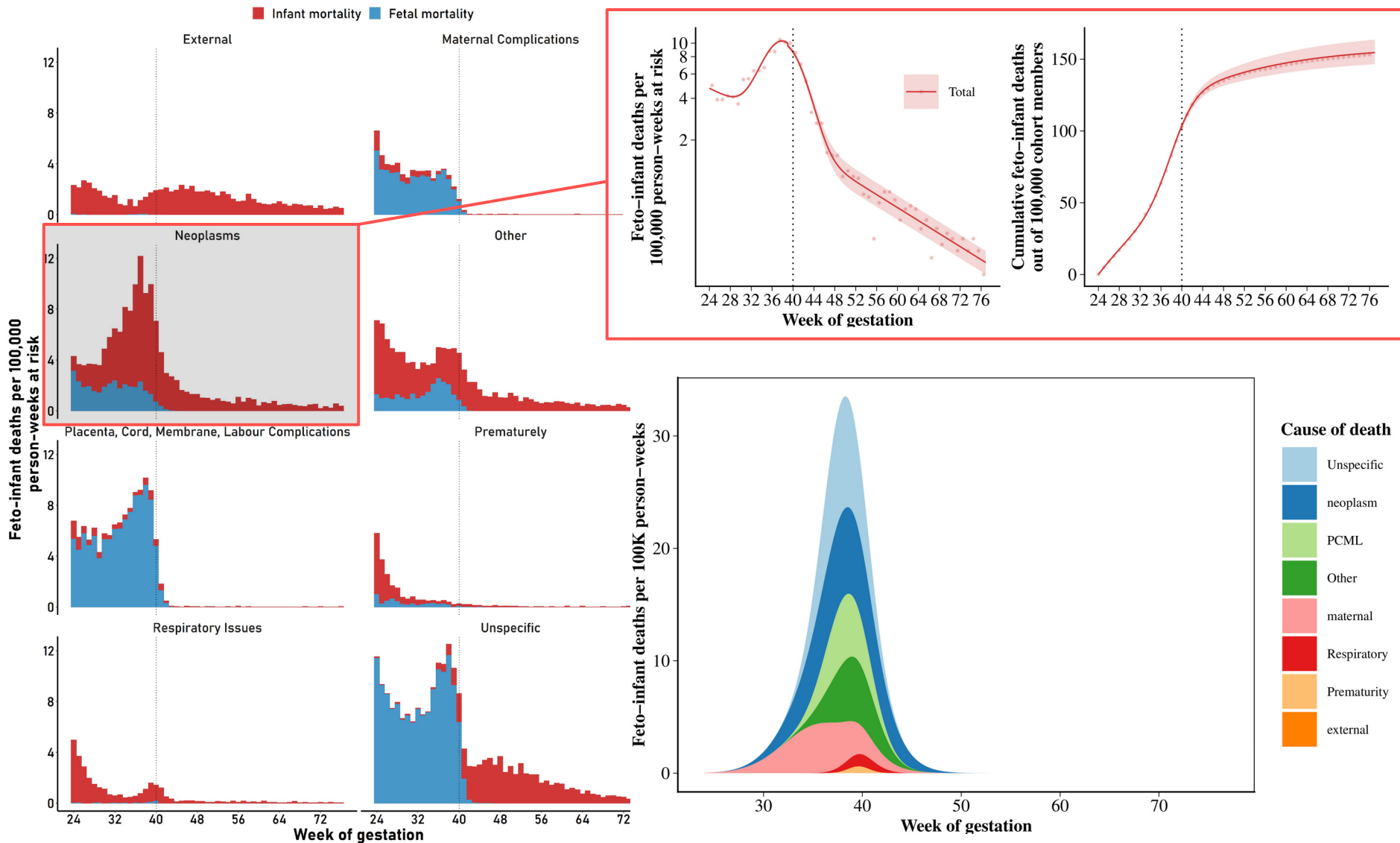
Data: US conception cohort 2014. CDC/NCHS.

# Cause of death composition of the “birth hump”



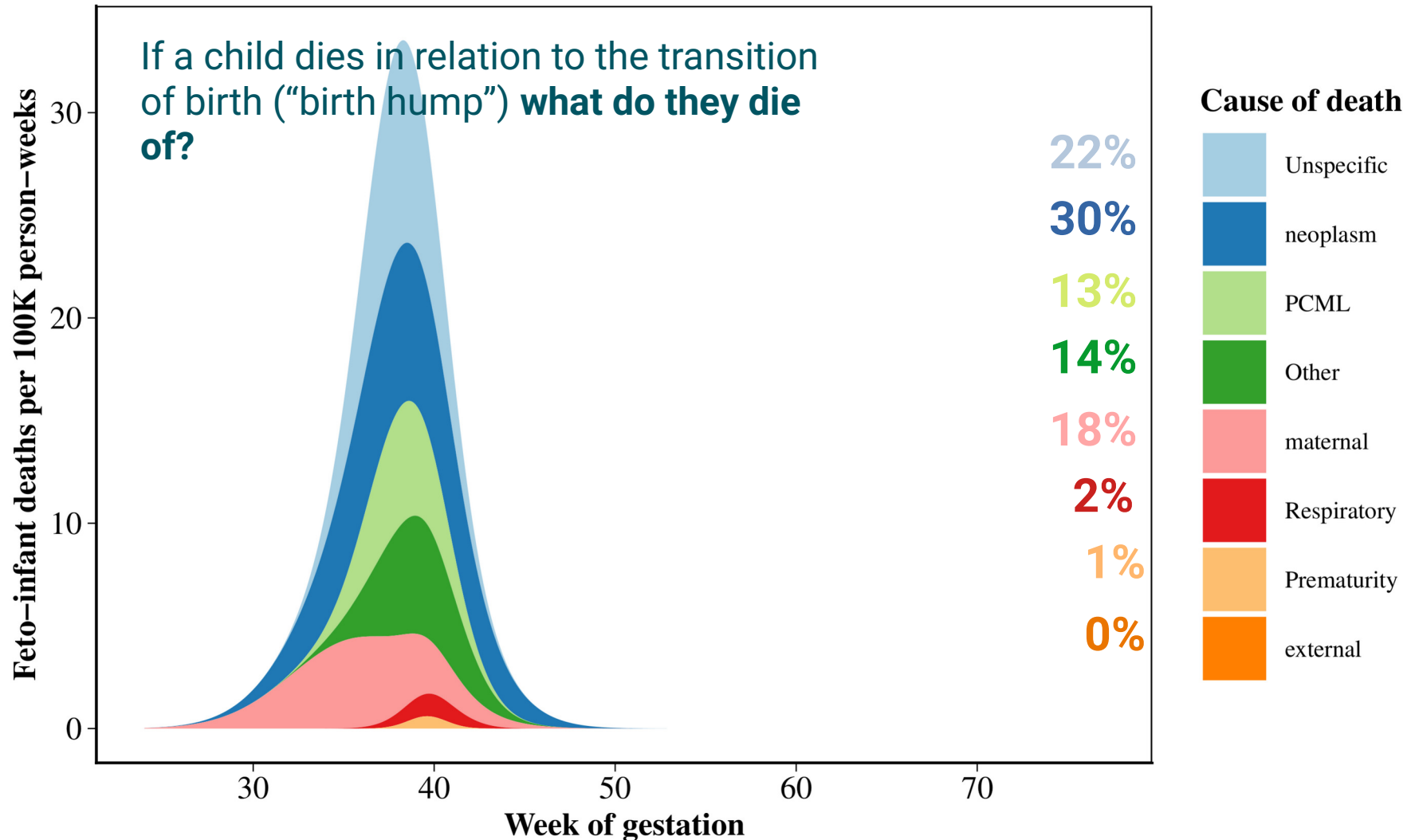
Data: US conception cohort 2014. CDC/NCHS.

# Cause of death composition of the “birth hump”



Data: US conception cohort 2014. CDC/NCHS.

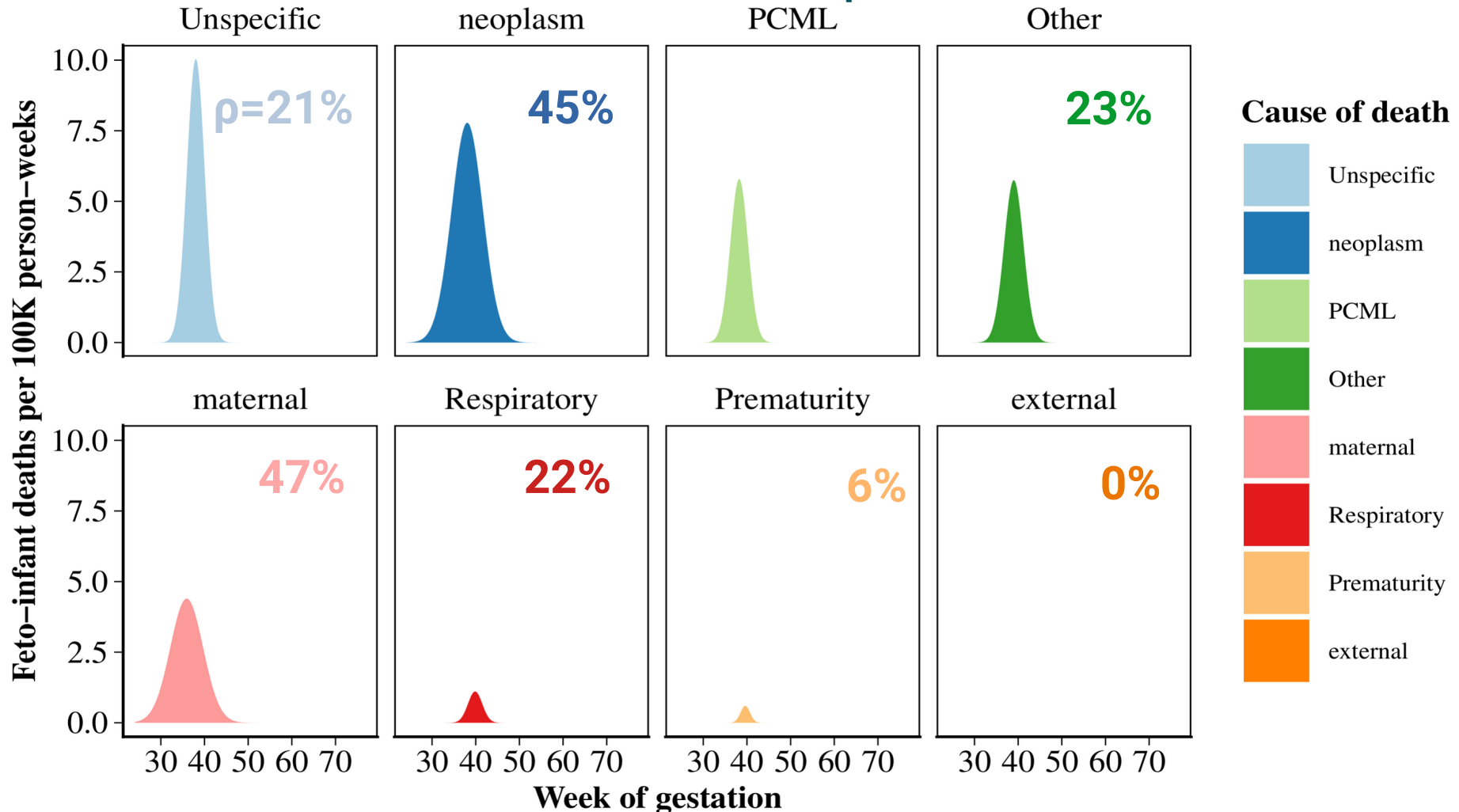
# Cause of death composition of the “birth hump”



Data: US conception cohort 2014. CDC/NCHS.

# Cause of death composition of the “birth hump”

Among those dying one year post fetal viability due to a given cause, **what share dies in relation to the birth hump?**



Data: US conception cohort 2014. CDC/NCHS.

# The transitional shock of birth

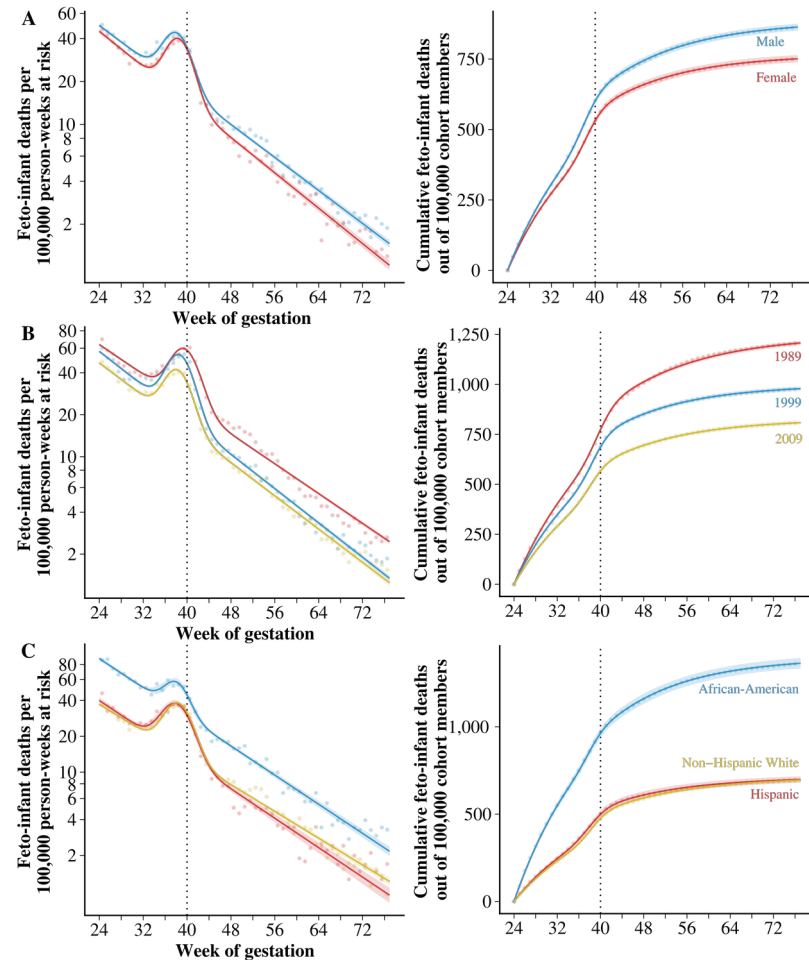
## The gestational age pattern of fetoinfant mortality

Jonas Schöley\*

### 1. Introduction

The different segments of a birth cohort's mortality trajectory have been thoroughly mapped starting with the sudden decline in the risk of death after a peak at birth (e.g., Bourgeois-Pichat 1951; Galley and Woods 1999; Berrut et al. 2016), the arrival at minimum risk in late childhood (Ebeling 2018), the “hump-shaped” excess mortality in adolescence (e.g., Thiele 1871; Goldstein 2011; Remund et al. 2018) and the exponential increase in the mortality hazard over much of the adult life (e.g., Gompertz 1825) which eventually flattens (e.g., Perks 1932; Vaupel 1997; Horiuchi and Wilmoth 1998) and then plateaus among the oldest-old (e.g., Gampe 2010; Barbi et al. 2018). Similar investigations have been made concerning the changing mortality risk of the unborn child over the age of a pregnancy (e.g., Shapiro et al. 1962; Bakketeig et al. 1978; Goldhaber and Fireman 1991; Carlson et al. 1999; Woods 2009).

Both survival scenarios, fetal and infant, meet at the point of birth but are nonetheless fundamentally separated by the use of different timescales. While prenatal mortality is indexed by gestational age, commonly measured as the weeks since the last menstrual period of the pregnant woman, the survival of those born alive is followed over chronological age, i.e., time since birth. Such a strict separation of populations along the dividing line of birth makes this critical transition invisible in the study of mortality, delegating to it either the role of a right censoring or a point of entry into the risk set. An alternative perspective allows bridging the fetoinfant gap by situating birth within the lifecycle of a cohort of unborn children whose survival is tracked over the age of gestation into infancy. By marking the vital events of fetal death, birth and infant death on a common age scale and its effect on the survival of a cohort on the onset of life can be studied by defining a fetoinfant mortality trajectory: the combined risk of fetal or infant death among all members of a conception cohort still alive at a given week of gestation.





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